

THE SOLUTION OF 3-DIMENSIONAL WAVE EQUATION IN CARTESIAN COORDINATE SYSTEM USING PARTIAL DIFFERENTIAL EQUATIONS

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ABSTRACT:

We study solutions of the 3-Dimensional wave equation with boundary Conditions on Cartesian co-ordinates, and we also study the analogous problem on a certain axis. This gives an alternative method of obtaining solutions of a corresponding problem in 3-Dimensional wave equation further, In this paper , we find the solution of 3-D wave equation .Three dimensional wave occur in earth quake ,tsunami and many physical states. In this paper we discussed the 3-D wave equation in XYZ axis and using partial differential equation

Keywords: 3-D Wave Equation, Partial Differential Equation, Fourier series.

INTRODUCTION:

Solutions of the wave equation with boundary conditions have many practical applications in engineering and physics. The paradigm of such textbook problems is that describing vibrations of a circular membrane (the shape of a drum) requiring solutions of the wave equation in a 3- dimensional. These solutions must vanish on the rectangular boundary of the membrane [8]..A theoretical application of much current interest, requiring such solutions, is the computation of sound energies for spherical boundary conditions. We show here that it is just as easy to set up such problems in a certain co-ordinate plane [3]

RESEARCH METHODOLOGY:

The following Research Methodology is adopted for the proposed Research paper:

- Identification of the problem
- Collection and study of related literature
- Mathematical formulation of the problem
- Analysis and numerical solution of the mathematical model
- Interpretation of results
- Conclusion

Mathematical formulation of the problem

The physical setting for our problem is as follows. We consider the three dimensional wave equations with the normal axis.

Three dimensional wave equation is

$$\frac{\partial^2 u}{\partial t^2} = \rho^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (1.1)$$

$$\text{Let } u(x, y, z, t) = XYZT \quad (1.2)$$

Where X is function of x only, Y is function of y only, Z is function of z only and T is function of t only.

$$\frac{\partial^2 u}{\partial t^2} = XYZT'', \frac{\partial^2 u}{\partial x^2} = X''YZT, \frac{\partial^2 u}{\partial y^2} = XY''ZT, \frac{\partial^2 u}{\partial z^2} = XYZ''T \quad \frac{\partial^2 u}{\partial t^2} = XYZT'' \quad (1.3)$$

From equation (1.1)

We have,

$$\frac{1}{\rho^2} XYZT'' = X''YZT + XY''ZT + XYZ''T$$

Or

$$\frac{1}{\rho^2} \frac{T''}{T} = \frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} \quad (1.4)$$

This will be true only when each member is a constant,

$$\frac{d^2 X}{dx^2} + k_1^2 X = 0, \frac{d^2 Y}{dy^2} + k_2^2 Y = 0, \frac{d^2 Z}{dz^2} + k_3^2 Z = 0 \text{ and } \frac{d^2 T}{dt^2} + (k_1^2 + k_2^2 + k_3^2) T = 0 \quad (1.5)$$

The solution of equation (1.5) are

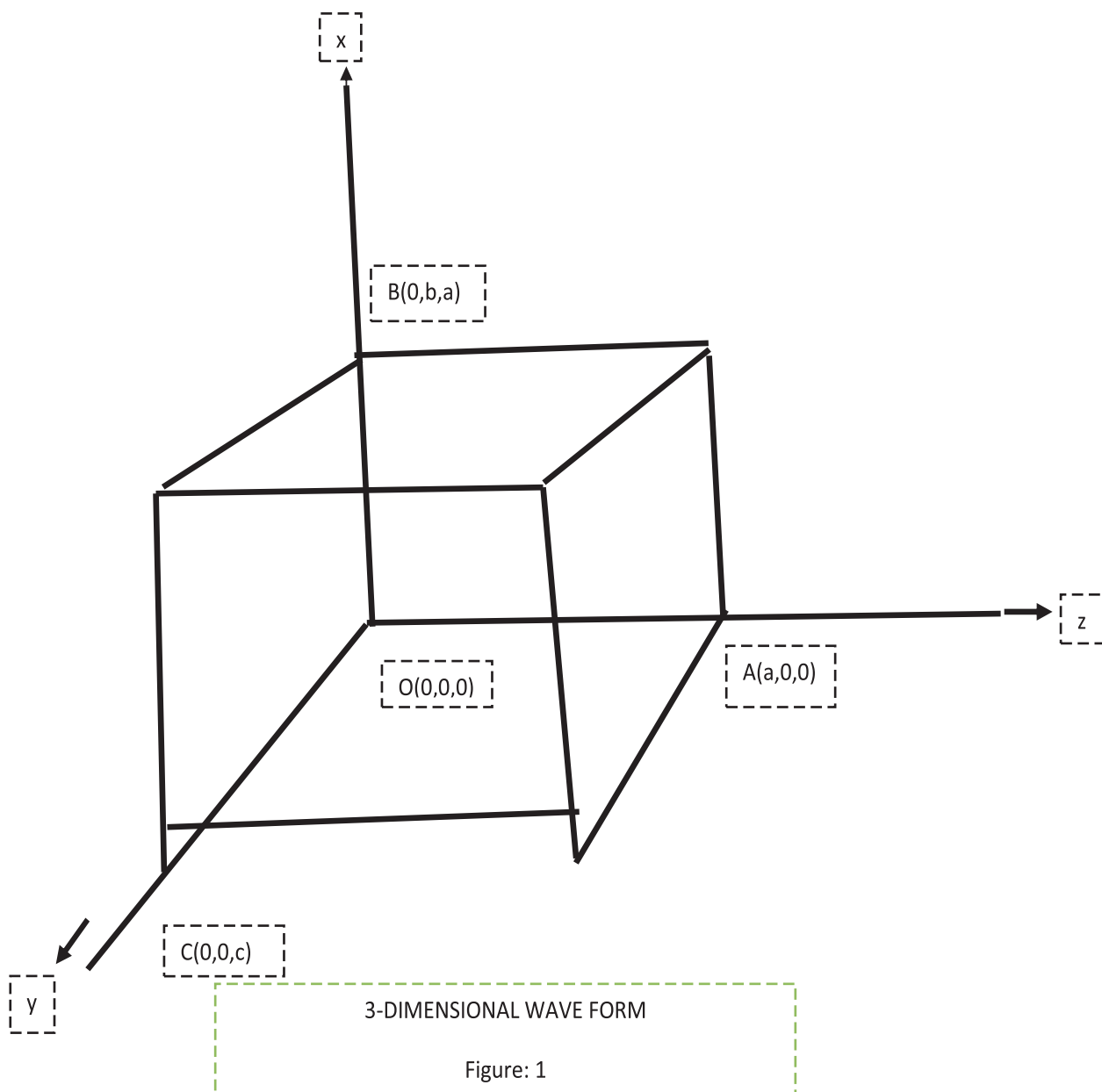
$$X = c_1 \cos k_1 x + c_2 \sin k_1 x, Y = c_3 \cos k_2 y + c_4 \sin k_2 y, Z = c_5 \cos k_3 z + c_6 \sin k_3 z \text{ and} \\ T = c_7 \cos \sqrt{(k_1^2 + k_2^2 + k_3^2)} \rho t + c_8 \sin \sqrt{(k_1^2 + k_2^2 + k_3^2)} \rho t \quad (1.6)$$

The solution of equation (1.1)

$$u(x, y, z, t) = (c_1 \cos k_1 x + c_2 \sin k_1 x) (c_3 \cos k_2 y + c_4 \sin k_2 y) (c_5 \cos k_3 z + c_6 \sin k_3 z) \\ (c_7 \cos \sqrt{(k_1^2 + k_2^2 + k_3^2)} \rho t + c_8 \sin \sqrt{(k_1^2 + k_2^2 + k_3^2)} \rho t) \quad (1.7)$$

Now Let us suppose that membrane is cuboids and stretched among the lines

$$x = 0, x = a; y = 0, y = b; z = 0, z = c$$



Then the boundary value conditions are :

- (i) $u(0, y, z, t) = 0$, when $x = 0$ (ii) $u(a, y, z, t) = 0$, when $x = a$ (iii) $u(x, 0, z, t) = 0$, when $y = 0$
 (iv) $u(x, b, z, t) = 0$, when $y = b$ (v) $u(x, y, 0, t) = 0$, when $z = 0$ (vi) $u(x, y, c, t) = 0$, when $y = c$
 for all t .

Applying condition (i) and (ii)

We have $c_1 = 0$ and $k_1 = \frac{l\pi}{a}$

$$u(x, y, z, t) = c_2 \sin \frac{l\pi x}{a} (c_3 \cos k_2 y + c_4 \sin k_2 y) (c_5 \cos k_3 z + c_6 \sin k_3 z) \\ (c_7 \cos \sqrt{(k_1 + k_2 + k_3)} \rho t + c_8 \sin \sqrt{(k_1 + k_2 + k_3)} \rho t) \quad (1.8)$$

Applying condition (iii) and (iv)

We have $c_3 = 0$ and $k_2 = \frac{m\pi}{b}$

$$u(x, y, z, t) = c_2 c_4 \sin \frac{l\pi x}{a} \sin \frac{m\pi y}{b} (c_5 \cos k_3 z + c_6 \sin k_3 z) \\ (c_7 \cos \sqrt{(k_1 + k_2 + k_3)} \rho t + c_8 \sin \sqrt{(k_1 + k_2 + k_3)} \rho t) \quad (1.9)$$

Now Applying condition (v) and (vi)

We have $c_5 = 0$ and $k_3 = \frac{n\pi}{c}$

$$u(x, y, z, t) = c_2 c_4 c_6 \sin \frac{l\pi x}{a} \sin \frac{m\pi y}{b} \sin \frac{n\pi z}{c} \\ (c_7 \cos \sqrt{(k_1 + k_2 + k_3)} \rho t + c_8 \sin \sqrt{(k_1 + k_2 + k_3)} \rho t) \quad (1.10)$$

$$u(x, y, z, t) = c_2 c_4 c_6 \sin \frac{l\pi x}{a} \sin \frac{m\pi y}{b} \sin \frac{n\pi z}{c} \\ \left(c_7 \cos \sqrt{\frac{l^2}{a^2} + \frac{m^2}{b^2} + \frac{n^2}{c^2}} \pi \rho t + c_8 \sin \sqrt{\frac{l^2}{a^2} + \frac{m^2}{b^2} + \frac{n^2}{c^2}} \pi \rho t \right) \quad (1.11)$$

Or

$$u(x, y, z, t) = c_2 c_4 c_6 \sin \frac{l\pi x}{a} \sin \frac{m\pi y}{b} \sin \frac{n\pi z}{c} (c_7 \cos pt + c_8 \sin pt) \quad (1.12)$$

Where

$$p = \sqrt{\frac{l^2}{a^2} + \frac{m^2}{b^2} + \frac{n^2}{c^2}} \pi \rho$$

General solution as

$$u(x, y, z, t) = \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin \frac{l\pi x}{a} \sin \frac{m\pi y}{b} \sin \frac{n\pi z}{c} (A_{lmn} \cos pt + B_{lmn} \sin pt) \quad (1.13)$$

Suppose the membranes start from the rest from the initial position

$$u(x, y, z, 0) = f(x, y, z)$$

$$\text{If } \left(\frac{\partial u}{\partial t} \right)_{t=0} = 0, \text{ we get } B_{lmn} = 0$$

$$f(x, y, z) = \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{lmn} \sin \frac{l\pi x}{a} \sin \frac{m\pi y}{b} \sin \frac{n\pi z}{c} \cos pt \quad (1.14)$$

$$\text{Using Fourier series, } \int_0^a \int_0^b \int_0^c f(x, y, z) \sin \frac{l\pi x}{a} \sin \frac{m\pi y}{b} \sin \frac{n\pi z}{c} dx dy dz = \frac{abc}{8} A_{lmn}$$

Or

$$A_{lmn} = \frac{8}{abc} \int_0^a \int_0^b \int_0^c f(x, y, z) \sin \frac{l\pi x}{a} \sin \frac{m\pi y}{b} \sin \frac{n\pi z}{c} dx dy dz$$

Thus the required solution of 3-D wave equation is

$$f(x, y, z, t) = \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{lmn} \sin \frac{l\pi x}{a} \sin \frac{m\pi y}{b} \sin \frac{n\pi z}{c} \cos \left(\sqrt{\frac{l^2}{a^2} + \frac{m^2}{b^2} + \frac{n^2}{c^2}} \right) \pi \rho t \quad (1.15)$$

$$\text{Where } A_{lmn} = \frac{8}{abc} \int_0^a \int_0^b \int_0^c f(x, y, z) \sin \frac{l\pi x}{a} \sin \frac{m\pi y}{b} \sin \frac{n\pi z}{c} dx dy dz$$

CONCLUSION

In this paper, for a general solution of three dimensional of wave equations is found and with the help of this solution, we have to find various kind of solution wave equations for example radio waves, telephonic wave etc.

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