Chapter 6
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The Onset of Multi-Diffusive Convection Analyzed for Suspended Particles in a Rotating Dusty Porous Layer: A Brinkman Model

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Abstract

Thermal convective instability of a horizontal layer of fluid heated from below has several applications in geophysics, earth science, and oceanography and extensive reviews of this subject can be found in Chandrasekhar [1]. The onset of multi-diffusive convection is analyzed to include the effects of suspended particles and rotation through a porous medium. In the present chapter, the Brinkman model is considered for the porous medium. The variations in fluid density are due to the variation in $(n+1)$ stratifying components having different thermal and solute diffusivities. Linear stability analysis procedure along with normal mode method is employed to obtain a dispersion relation for the stationary convection and it is found that the parameters porosity, permeability and suspended particle have destabilizing effects whereas, rotation and Darcy-Brinkman number have stabilizing effects and the results are also shown both numerically and graphically. A sufficient condition for the validity of the principle of exchange of stabilities (PES) is also obtained using Rayleigh-Ritz and Cauchy-Schwartz inequality.

Keywords: Multi-diffusive convection; suspended particles; rotation; Brinkman porous medium.

Nomenclature Used

t	Time co-ordinate, $[s]$
d	Depth of fluid layer, $[m]$

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$\mathbf{q}$	Velocity of fluid having components (u,v,w) , $\left[ms^{-1}\right]$
p	Pressure, $\left[Nm^{-2}\ or\ Pa\right]$
T_0	Reference temperature, $[K]$
T	Temperature, $[K]$
k_1	Darcy-Brinkman medium permeability, $\left[m^2\right]$
P_l	Dimensionless medium permeability, $[-]$
k_T	Coefficient of heat conduction, $\left[Wm^{-1}K^{-1}\right]$
D	Differentiation Operator $\left(=\frac{d}{dz}\right)$, $[-]$
n	Frequency of the harmonic disturbance, $\left[s^{-1}\right]$
$\mathbf{X_i}$	Gravitational acceleration vector $(=-g\lambda_i)$, $\left[ms^{-2}\right]$
c_s	Heat capacity of solid material, $\left[Jkg^{-1}K^{-1}\right]$
c_v	Specific heat of the fluid at constant volume, $\left[Jkg^{-1}K^{-1}\right]$
p_1	Thermal Prandtl number, $[-]$
p_2	Magnetic Prandtl number, $[-]$
$\mathbf{w}$	Vertical fluid velocity, $\left[ms^{-1}\right]$
$\mathbf{w}^*$	Complex conjugate of $\mathbf{w}$
$\mathbf{H}$	Horizontal magnetic field having components $(H,0,0)$, $[G]$
$\mathbf{h}$	Perturbation in magnetic field strength $\mathbf{H}(H,0,0)$, $[G]$
$\mathbf{X}$	Vertical component of current density after applying the normal mode method
$\mathbf{W}$	Vertical component of fluid velocity after applying normal mode method
$\mathbf{K}$	Vertical component of the magnetic field after applying the normal mode method
$\mathbf{Z}$	Vertical component of vorticity after applying normal mode method
k_x	Wave number in x direction, $\left[m^{-1}\right]$
k_y	Wave number in y direction, $\left[m^{-1}\right]$
k	Resultant wave number $\left\{=\sqrt{\left(k_x{}^2+k_y{}^2\right)}\right\}$, $\left[m^{-1}\right]$
Q_1	Modified Chandrasekhar's number, $[-]$
D_{A_1}	Modified Darcy-Brinkman number, $[-]$

R_1 Modified Darcy-Brinkman thermal Rayleigh number, $[-]$

T_{A_1} Modified Taylor's number, $[-]$

Greek Symbols

∇p Pressure gradient term $\left[Pa\,m^{-1}\right]$

$\in$ Darcy-Brinkman medium porosity

ρ_0 Density of fluid, $\left[kgm^{-3}\right]$

ρ_s Density of solid material, $\left[kgm^{-3}\right]$

μ Fluid viscosity, $\left[kgm^{-1}s^{-1}\right]$

μ' Couple-stress fluid viscosity $\left[kgm^{-1}s^{-1}\right]$

μ_{ef} Effective viscosity $\left[kgm^{-1}s^{-1}\right]$

μ_e Magnetic permeability, $\left[Hm^{-1}\right]$

∂ Curl Operator, $[-]$

α Co-efficient of thermal expansion, $\left[K^{-1}\right]$

β Adverse temperature gradient, $\left[Km^{-1}\right]$

η Electrical resistivity, $\left[m^2s^{-1}\right]$

Θ temperature component after applying the normal mode method

δp Perturbation in fluid pressure p, $\left[Nm^{-2}\ or\ Pa\right]$

$\delta\rho$ Perturbation in fluid density ρ , $\left[kgm^{-3}\right]$

υ Kinematic viscosity, $\left[m^2s^{-1}\right]$

υ' Kinematic viscoelasticity, $\left[m^2s^{-1}\right]$

κ Thermal diffusivity, $\left[m^2s^{-1}\right]$

ζ Z-component of vorticity

ξ Z-component of current density

$\mathbf{\Omega}$ Horizontal rotational vector

∇^2 3-dimensional Laplacian operator, $[-]$

Υ_1 Modified couple-stress parameter, $[-]$

σ Growth rate of harmonic disturbance after applying the normal mode method, $\left[s^{-1}\right]$

θ Perturbation in temperature T, $[K]$

λ_i Vertical unit vector, $[-]$

1 Introduction

Convective instability occurs in foam under forced drainage when a critical liquid fraction is exceeded [2]. Thermal convective instability of a horizontal layer of fluid heated from below has several applications in geophysics, earth science, and oceanography and extensive reviews of this subject can be found in Chandrasekhar [1]. Rayleigh [3] laid the foundation of the linear instability theory using small infinitesimal perturbations. When two or more stratifying components (e.g. heat and salt diffusing at different rates) are present then the convective phenomenon is termed as Double-diffusive or Multi-diffusive convection having extensive physical applications in ocean water, magmas, contaminant transport and underground water flow [4,5,6]. The investigation of stratified fluid layer has several applications in thermal stratification of reservoirs and oceans, density, temperature and gravitational stratification of the atmosphere, salinity stratification in rivers, oceans and estuaries, layer stratification in the earth's interior and several heterogeneous mixtures in food processing industry [7]. Magneto-hydrodynamics theory of electrically conducting fluids has several scientific and practical applications in astrophysics, geophysics, space sciences etc [8].

The flow through a porous medium has been of fundamental importance in geothermal reservoirs, solidification, geothermal power resources, astrophysics, the chemical processing industry, the petroleum industry, and the recovery of crude oil from the earth's interior [9,10]. A detailed study of convection through a porous layer can be found in Nield and Bejan [11]. The numerical and analytical treatment of the double-diffusive and multi-diffusive convection saturating a porous layer is reviewed in the references Huppert and Turner [12], Turner [13], Terrones and Pearlstein [14], Tracey [15], Straughanand Tracey [16], Radko [17, Rionero [18,19], Prakash et al. [20], Kumar et al. [21].

Convective instability in a rotating frame has numerous applications in rotating machinery, the food processing industry, centrifugal casting of metals and thermal power plants (to generate electricity by rotation of turbine blades) [6]. Rudraiah et al. [22] considered the effect of rotation on linear and non-linear double-diffusive convective problems saturating a porous layer.

In a geophysical context, the fluid is often not pure but may instead be permeated with dust particles. These suspended particles have scientific relevance in geophysics, chemical engineering and astrophysics [23]. Scanlon and Segel [24] considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure fluid was supplemented by that of the particles.

The intention of the present chapter is to analyse the onset of thermal convection in a multi-diffusive fluid layer in the presence of suspended dust particles, uniform vertical rotation saturating a porous medium. Most research outcomes for porous medium flows are based on the Darcy model which gives appropriate results at a small Reynolds number. Therefore, the Darcy-Brinkman model is employed for porous medium which is considered physically more realistic than the usual Darcy model and also gave satisfactory results at large Reynolds number and for high porosity porous medium by incorporating the inertial and viscous effects in addition to the usual Darcy model. The research on multi-component fluid layers through a porous medium has notable geophysical relevance in real life and is increasing with the number of salts dissolved in it.

2 Problem Formulation and Linear Stability Analysis

Consider an infinite horizontal Boussinesq fluid layer permeated with dust particles lying in the region $0 \le z \le d$ through a Darcy-Brinkman porous medium under the effect of a uniform vertical rotation $\mathbf{\Omega}(0,0,\Omega_z)$. Both the boundaries are maintained at uniform temperatures $T_l(>T_u)$, T_u and uniform n concentrations $C_l^{\,1}\left(>C_u^{\,1}\right), C_l^{\,2}\left(>C_u^{\,2}\right),........,C_l^{\,n}\left(>C_u^{\,n}\right)$ and $C_u^{\,1}, C_u^{\,2},......,C_u^{\,n}$ with gravity acting in the vertical downward direction (Fig. 1) [6].

The governing equations of motion and continuity for an incompressible Oberbeck-Boussinesq [25] fluid layer saturating a Darcy-Brinkman porous medium [26,27] are as:

$$\frac{1}{\in}\left[\frac{\partial v}{\partial t}+\frac{1}{\in}\left(v\frac{\partial v}{\partial x}\right)\right]=\begin{bmatrix}-\frac{1}{\rho_0}(\nabla p)-\frac{\upsilon}{k_1}v+\tilde{\upsilon}_{ef}\left(\nabla^2 v\right)+\\ \left(1+\frac{\delta\rho}{\rho_0}+\frac{\delta\rho_1}{\rho_0}+\frac{\delta\rho_2}{\rho_0}+.......+\frac{\delta\rho_n}{\rho_0}\right)\mathbf{g}\\ +\frac{K' N_0}{\rho_0 \in}(v_d-v)+\frac{2}{\in}(v\times\mathbf{\Omega})\end{bmatrix} \tag{1}$$

$$\nabla.v = 0 \tag{2}$$

Where, $t, \rho_0, \rho, \in, p, \upsilon, \tilde{\upsilon}_{ef}, v, v_d, k_1, N_0$ *and* $\mathbf{g}$ denote, respectively, the time, the reference density, fluid density, effective porosity, pressure, kinematic viscosity, effective kinematic viscosity, fluid velocity components, particles velocity, effective permeability, number density of suspended particles and the gravitational acceleration vector. The term $K' = 6\pi\rho\upsilon\delta$ (δ being particle radius), is the Stokes drag coefficient.

The presence of suspended particles adds an extra force term, in the equation of motion, proportional to the velocity difference between particles and fluid [6]. Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. Inter-particle reactions are ignored as the distances between the particles are assumed to be quite large compared with their diameters.

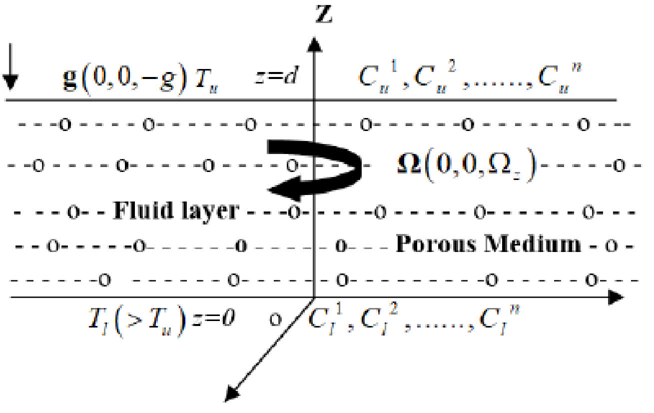

Fig. 1. Geometrical sketch of the physical problem

The equations of motion and continuity for the particles (ignoring the pressure, magnetic field and gravity) are as:

$$mN_0\left[\frac{\partial v_d}{\partial t}+\frac{1}{\in}(v_d.\nabla)v_d\right]=K'N_0(v-v_d) \tag{3}$$

$$\in\frac{\partial N_0}{\partial t}+\nabla.(N_0v_d)=0 \tag{4}$$

where mN_0 is the mass of particles per unit volume.

The equations for temperature field and solute concentrations are as:

$$\begin{aligned}&\left[\in\rho_0c_v+\rho_sc_s(1-\in)\right]\frac{\partial T}{\partial t}+\rho_0c_v(v.\nabla)T\\&+mN_0c_{pt}\left(\in\frac{\partial}{\partial t}+v_d.\nabla\right)T=k_T\nabla^2T\end{aligned} \tag{5}$$

where, c_v denote, respectively, the density of solid material, the heat capacity of solid material, the specific heat at constant volume, the heat capacity of suspended particles, the temperature and the coefficient of heat conduction.

$$\left[\in \rho_0 c_v{}^{\lambda} + \rho_s c_s{}^{\lambda}\left(1-\in\right)\right]\frac{\partial C^{\lambda}}{\partial t} + \rho_0 c_v{}^{\lambda}\left(v.\nabla\right)C^{\lambda}$$
$$+mN_0 c_{pt}{}^{\lambda}\left(\in\frac{\partial}{\partial t} + v_d.\nabla\right)C^{\lambda} = k_{C^{\lambda}}\nabla^2 C^{\lambda}\ \left(\lambda = 1,2,...,n\right) \tag{6}$$

The symbols $c_s{}^{\lambda}, c_v{}^{\lambda}, c_{pt}{}^{\lambda}, C^{\lambda}$ *and* $k_C{}^{\lambda}\left(\lambda = 1,2,,...,n\right)$ denote the analogous n solute components.

The density is taken as a linear function of the temperature field and salt concentrations as:

$$\rho = \rho_0\left[1 + \alpha_T\left(T_l - T_u\right) - \sum_{\lambda=1}^{n}\alpha_{C^{\lambda}}\left(C_l{}^{\lambda} - C_u{}^{\lambda}\right)\right] \tag{7}$$

where, $T_l, T_u, \alpha_T, \alpha_{C_{\lambda}}, C_l{}^{\lambda}$ *and* $C_u{}^{\lambda}$ $\left(\lambda = 1,2,.....n\right)$ denote, respectively, the temperature at the lower boundary, the temperature at the upper boundary, the coefficient of thermal expansion, coefficients of solute expansion, and concentration components at the lower and upper boundaries.

The basic state is assumed to be stationary and therefore, for determining the stability/instability of the system linear stability analysis procedure followed by the normal mode method is adopted by introducing small infinitesimal perturbations in the basic variables (Singh et al., 2022).

The basic state of the system is defined as:

$$v = (0,0,0),\ v_d = (0,0,0), T = T_0 - \beta_T z,\ \Omega = \left[0,0,\Omega\right],$$
$$\rho = \rho_0\left[1 + \alpha\beta_T z\right],\ p = p_0 - g\rho_0 z\left(1 + \frac{\alpha\beta_T z}{2}\right),\ N_0,\ C^{\lambda}. \tag{8}$$

Let the perturbations in the basic variables given in (8) are defined as:

$$v = (u,v,w), v_d = (l,r,s), \theta, \Omega\left(\Omega_x, \Omega_y, \Omega_z\right), \delta\rho, \delta p, N, \gamma^{\lambda}. \tag{9}$$

So, the resulting linearized perturbation equations after eliminating the pressure gradient term are as:

$$\frac{1}{\in}\frac{\partial}{\partial t}\left(\nabla^2 w\right)=\begin{bmatrix}-\frac{\upsilon}{k_1}\left(\nabla^2 w\right)+\tilde{\upsilon}_{ef}\left(\nabla^4 w\right)+g\alpha_T{\nabla_1}^2\theta \\ -g{\nabla_1}^2\sum_{\lambda=1}^{n}\alpha_{C^\lambda}\gamma^\lambda-\frac{mN_0}{\rho_0\in\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)} \\ \frac{\partial}{\partial t}\left(\nabla^2 w\right)-\frac{2\Omega}{\in}\left(\frac{\partial\zeta}{\partial z}\right)\end{bmatrix} \tag{10}$$

$$\frac{1}{\in}\left(\frac{\partial\zeta}{\partial t}\right)=\begin{bmatrix}-\frac{\upsilon}{k_1}\zeta+\tilde{\upsilon}_{ef}\left(\nabla^2\zeta\right)-\frac{mN_0}{\rho_0\in\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)} \\ \left(\frac{\partial\zeta}{\partial t}\right)+\frac{2\Omega}{\in}\left(\frac{\partial w}{\partial z}\right)\end{bmatrix} \tag{11}$$

$$\left[\left(E+b\in\right)\frac{\partial}{\partial t}-\kappa_T\nabla^2\right]\theta=\beta_T\left[1+\frac{b}{\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)}\right]w \tag{12}$$

$$\left[\left(E^\lambda+b^\lambda\in\right)\frac{\partial}{\partial t}-\kappa_{C^\lambda}\nabla^2\right]\gamma^\lambda=\beta_{C^\lambda}\left[1+\frac{b^\lambda}{\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)}\right]w \tag{13}$$

The change in density $\delta\rho$ due to temperature variation θ and concentration variations $\gamma^\lambda\left(\lambda=1,2,....,n\right)$, is given by

$$\delta\rho=-\left(\alpha_T\theta-\sum_{\lambda=1}^{n}\alpha_{C^\lambda}\gamma^\lambda\right)\rho_0 \tag{14}$$

where, in equations (10)-(13), $\kappa_T=\frac{k_T}{\rho_0 c_v},\kappa_{C^\lambda}=\frac{k_{C^\lambda}}{\rho_0 {c_v}^\lambda},w,\ s,\zeta\left(=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right),{\nabla_1}^2 and\ \nabla^2$ denote, respectively, the thermal diffusivity, the solute diffusivity, the vertical component of fluid velocity, the vertical component of suspended particles velocity, the vertical component of vorticity, the horizontal Laplacian operator and Laplacian operator, with

$$E = \in + (1-\in)\left(\frac{\rho_s c_s}{\rho_0 c_v}\right), b = \frac{mN_0 c_{pt}}{\rho_0 c_v},$$
$$E^{\lambda} = \in + (1-\in)\left(\frac{\rho_s {c_s}^{\lambda}}{\rho_0 {c_v}^{\lambda}}\right) and\ b^{\lambda} = \frac{mN_0 {c_{pt}}^{\lambda}}{\rho_0 {c_v}^{\lambda}}.$$

3 Normal Mode Method and Dispersion Relation

A normal mode representation is assumed in various physical disturbances with a dependence on $x, y\ and\ t$ of the form:

$$\left[w, \theta, \zeta, \gamma^{\lambda}\right] = \left[W(z), \Theta(z), Z(z), \Phi^{\lambda}(z)\right] \exp\left(ik_x x + ik_y y + nt\right) \tag{15}$$

where, $k_x\ and\ k_y$ are the wave numbers along $x\ and\ y$ directions, respectively.

Using expression (15), the non-dimensional form of Eqs. (10) -(13) (after dropping the asterisk for convenience) are as:

$$\left[\frac{\sigma}{\in}\left\{1 + \frac{M}{(1+\tau_1\sigma)}\right\} - \frac{D_A}{P_l}\left(D^2 - a^2\right) + \frac{1}{P_l}\right]\left(D^2 - a^2\right)W + \frac{ga^2d^2}{\upsilon}\left[\alpha_T\Theta - \sum_{\lambda=1}^{n}\alpha_{C^{\lambda}}\Phi^{\lambda}\right] + \frac{2\Omega d^3}{\in \upsilon}DZ = 0 \tag{16}$$

$$\left[\frac{\sigma}{\in}\left(1 + \frac{M}{(1+\tau_1\sigma)}\right) - \frac{D_A}{P_l}\left(D^2 - a^2\right) + \frac{1}{P_l}\right]Z + \frac{2\Omega d}{\in \upsilon}DW = 0 \tag{17}$$

$$\left[\left(D^2 - a^2\right) - p_1 E_1 \sigma\right]\Theta = -\left(\frac{\beta_T d^2}{\kappa_T}\right)\left(\frac{B + \tau_1\sigma}{1+\tau_1\sigma}\right)W \tag{18}$$

$$\left[\left(D^2 - a^2\right) - q^{\lambda} {E_1}^{\lambda}\sigma\right]\Theta^{\lambda} = -\left(\frac{\beta_{C^{\lambda}} d^2}{\kappa_{C^{\lambda}}}\right)\left(\frac{B^{\lambda} + \tau_1\sigma}{1+\tau_1\sigma}\right)W \tag{19}$$

The above perturbation equations (16)-(19) are non-dimensionalized using the following scaling:

$$z^* = \left(\frac{z}{d}\right), k = \left(\frac{a}{d}\right), \sigma = \frac{nd^2}{\upsilon}, \tau = \frac{m}{K'}, \tau_1 = \frac{\tau\upsilon}{d^2}, B = 1 + b,$$
$$B^{\lambda} = 1 + b^{\lambda}, N_0 = \frac{\rho_0 M}{m}, D_A = \left(\frac{\tilde{\mu}_{ef} P_l}{\mu}\right), E_1 = E + b\in,$$
$${E_1}^{\lambda} = E^{\lambda} + b^{\lambda}\in, P_l = \frac{k_1}{d^2}, p_1 = \frac{\upsilon}{\kappa_T}, q^{\lambda} = \frac{\upsilon}{\kappa_{C^{\lambda}}}.$$

where, P_l is the dimensionless medium permeability, p_1 is the thermal Prandtl number, $q^{\lambda}\,(\lambda = 1, 2, ..., n)$ are the n Schmidt numbers, β_T is the adverse temperature gradient, $\beta_{C^\lambda}\left(\lambda=1,2,...,n\right)$ are the n solute concentration gradients, $k^2=\left(k_x{}^2+k_y{}^2\right)$ is a wave number and n is the frequency of the harmonic disturbance and $D=\left(\dfrac{d}{dz}\right)$.

The boundary conditions (for the case of two free boundaries are defined as:

$$W=D^2W=DZ=\Theta=\Phi^\lambda\left(\lambda=1,2,..,n\right)=0 \; at \; z=0 \text{ and d.} \tag{20}$$

Eliminating $\Theta(z)$, $\Phi^\lambda(\mathbf{z})$ *and* $\mathbf{Z}(\mathbf{z})$ from equations (16)–(19), a dispersion relation in W is obtained as:

$$\begin{aligned}&\left[\frac{\sigma}{\in}\left\{1+\frac{M}{\left(1+\tau_1\sigma\right)}\right\}-\frac{D_A}{P_l}\left(D^2-a^2\right)+\frac{1}{P_l}\right]\left(D^2-a^2\right)W\\&-\frac{R_Ta^2\left(\dfrac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)}{\left[\left(D^2-a^2\right)-p_1E_1\sigma\right]}W+\frac{R_{C^\lambda}a^2\left(\dfrac{B^\lambda+\tau_1\sigma}{1+\tau_1\sigma}\right)}{\left[\left(D^2-a^2\right)-q^\lambda E_1{}^\lambda\sigma\right]}W\\&-\frac{T_A}{\in^2\left[\dfrac{\sigma}{\in}\left\{1+\dfrac{M}{\left(1+\tau_1\sigma\right)}\right\}-\dfrac{D_A}{P_l}\left(D^2-a^2\right)+\dfrac{1}{P_l}\right]}D^2W=0\end{aligned} \tag{21}$$

where, $R_T=\dfrac{g\alpha_T\beta_Td^4}{\upsilon\kappa_T}$ (thermal Rayleigh number), $R_{C^\lambda}=\dfrac{g\alpha_{C^\lambda}\beta_{C^\lambda}d^4}{\upsilon\kappa_{C^\lambda}}$ (solute Rayleigh numbers), $T_A=\dfrac{4\Omega^2d^4}{\upsilon^2}$ (Taylor number).

4 The Stationary Convection

For stationary state $(\sigma=0)$, Eq. (21) yields an expression of the form:

$$\begin{aligned}&\left[1-D_A\left(D^2-a^2\right)\right]^2\left(D^2-a^2\right)^2W-\left[R_TB-\sum_{\lambda=1}^{n}R_{C^\lambda}B^\lambda\right]\\&\left[1-D_A\left(D^2-a^2\right)\right]a^2P_lW-\frac{T_AP_l{}^2}{\in^2}\left(D^2-a^2\right)D^2W=0\end{aligned} \tag{22}$$

Since all the even derivatives of W vanishes, so considering an appropriate solution for W of the form:

$W = W_0 \sin l\pi z,\ (W_0 \neq 0,\ l = 1,2,3....)$

Equation (22) yields:

$$R_T{}^{\dagger} = \frac{1}{B}\left[\begin{array}{l}\displaystyle\sum_{\lambda=1}^{n} R^{\dagger}{}_{C^{\lambda}}\, B^{\lambda} + \frac{\{1 + D_{A_1}(1+x)\}(1+x)^2}{Px} \\ \displaystyle -\frac{T_{A_1}P(1+x)}{\in^2 x\{1 + D_{A_1}(1+x)\}}\end{array}\right] \tag{23}$$

where, the following notations are assumed as:

$R_T{}^{\dagger} = \frac{R_T}{\pi^4}, R^{\dagger}{}_{C^{\lambda}} = \frac{R_{C^{\lambda}}}{l^4\pi^4}, x = \frac{a^2}{l^2\pi^2}, P_l = \frac{P}{l^2\pi^2},$ Minimizing Eq. (23) with respect $T_{A_1} = \frac{T_A}{l^4\pi^4}, D_{A_1} = \frac{D_A}{l^2\pi^2}.$
to $x\left(i.e. \frac{\partial R_T{}^{\dagger}}{\partial x} = 0\right)$ yields a fifth degree equation in x as:

$$\alpha_1 x^5 + \alpha_2 x^4 + \alpha_3 x^3 + \alpha_4 x^2 + \alpha_5 x + \alpha_6 = 0 \tag{24}$$

where,

$$\begin{aligned}
&\alpha_1 = \left(2 \in^2 D_{A_1}{}^3\right), \alpha_2 = \left(7 \in^2 D_{A_1}{}^3 + 5 \in^2 D_{A_1}{}^2\right),\\
&\alpha_3 = \left(8 \in^2 D_{A_1}{}^3 + 12 \in^2 D_{A_1}{}^2 + 4 \in^2 D_{A_1}\right),\\
&\alpha_4 = \left(2 \in^2 D_{A_1}{}^3 + 6 \in^2 D_{A_1}{}^2 + 5 \in^2 D_{A_1} + T_{A_1}P^2 D_{A_1} + \in^2\right),\\
&\alpha_5 = \left(-2 \in^2 D_{A_1}{}^3 - 4 \in^2 D_{A_1}{}^2 - 2 \in^2 D_{A_1} + 2T_{A_1}P^2 D_{A_1}\right),\\
&\alpha_6 = \left(- \in^2 D_{A_1}{}^3 - 3 \in^2 D_{A_1}{}^2 - 3 \in^2 D_{A_1} - \in^2 + T_{A_1}P^2 D_{A_1} + T_{A_1}P^2\right).
\end{aligned}$$

The critical dimensionless wave number x_c for varying values of parameters can be obtained from Eq. (24) and then the critical thermal and solute Rayleigh numbers can be deduced from Eq. (23) [6].

Equation (23) represents a relationship between thermal and solute Rayleigh numbers in terms of various embedded parameters. The effect of these parameters (suspended particles, medium permeability, medium porosity, Taylor number, Darcy-Brinkman) on thermal Rayleigh number can be examined analytically from the following derivatives $\frac{dR_T{}^{\dagger}}{dB}, \frac{dR_T{}^{\dagger}}{dB^{\lambda}}, \frac{dR_T{}^{\dagger}}{dP}, \frac{dR_T{}^{\dagger}}{d\in}, \frac{dR_T{}^{\dagger}}{dT_{A_1}}$ and $\frac{dR_T{}^{\dagger}}{dD_{A_1}}$.

5 Conclusion

(a) A linear stability analysis followed by the normal mode method is taken into account to discuss the effect of uniform vertical rotation and suspended particles on the onset of multi-diffusive convection through a Darcy-Brinkman porous medium. It is concluded that for the stationary convection,

i. The suspended particles, medium porosity and medium permeability are found to hasten the onset of thermal instability when the gravity field increases upward from its value g_0 i.e. $\left(f\left(z\right)>0\right)$.

ii. The effects of the magnetic field and couple-stress parameter are to stabilize the system, as such their effect is to postpone the onset of thermal instability when the gravity field increases upward from its value g_0 i.e. $\left(f\left(z\right)>0\right)$.

(b) For the validity of PES, the necessary condition for the onset of instability is that the inequality given in Eq. (50) must be satisfied.

(c) In the absence of a couple-stress parameter (i.e. $\Upsilon = 0$), the necessary condition for the onset of instability is that the inequality $R > \frac{4\pi^2}{P_l}$ is satisfied and thus the sufficient condition for the non-existence of stability is that $R \le \frac{4\pi^2}{P_l}$.

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Competing Interests

Authors have declared that no competing interests exist.

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