Chapter 3
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Convectional Onset in Ferrofluid Layer Through a Darcy-brinkman Porous Medium

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ABSTRACT

This chapter considers the analysis of the instabilities of convection through a porous medium in a layer of incompressible ferrofluid when the layer is subjected to a uniform magnetic field along with an external heat source. The Darcy-Brinkman model is used to study porous media. The perturbation method is used in combination with the normal mode method to analyse the influence of various integrated factors involved in the stability/instability of the considered system. The case of exponential variation in stratification is considered, and the dependence of the growth rate on the kinematic viscosity, medium porosity, medium permeability, heat source, Darcy-Brinkman number, and Alfvén squared velocity is also analytically demonstrated. The cases of stable stratification and unstable stratification are also analysed to determine the stabilisation as well as destabilisation effects of the system under certain constraints.

Keywords: Ferrofluids; magnetic field; Darcy-Brinkman porous medium; heat source.

1. INTRODUCTION

A comprehensive and detailed overview of thermal instability problems of an incompressible Newtonian fluid under the simultaneous effects of rotation and magnetic field has been given by Chandrasekhar (1981) in his famous monograph. The study of non-Newtonian fluids is of significant importance with that of Newtonian fluids. One such industrially important fluid is known as *ferrofluid*. Ferro fluid, which is also known as magnetic solution, is a non-

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electrical colloid that has a wide range of applications in actuators, acoustics, lubrication, loudspeakers, magnetic fluid bearings, metal recovery, sensors, targeted drug delivery, tumour detection, vacuum technology and many more (Hathaway, 1979; Hassan & Pop, 2021). Finlayson (1970) considered the convective instability of a layer of ferrofluid heated from below under the influence of a uniform vertical magnetic field to investigate the effect on the layer with and without the body force (gravity) influences. A comprehensive overview of ferro-hydrodynamics has been given by Rosensweig (1985) and Odenbach (2002). The investigation of stratified fluid layers has several applications in different types of stratification associated with reservoirs, estuaries, rivers, oceans, atmosphere, Earth's interior, etc. (Penfield & Haus, 1967; Sheikholeslami, 2023; Siddiqa& Mahapatra, 2020).

The magnetohydrodynamic theory of conducting fluids has various scientific and practical applications in astrophysics, geophysics, space science, and other fields (Vajravelu&Sreenadh, 2018). Thermal and thermosolutal instability problems of a ferromagnetic fluid under the effects of a magnetic field and dust particles saturating a porous medium have been studied by Sunil et al. (2004, 2005). In particular, the strength of the heat source plays an important role in the actual understanding of many of the heat transfer processes like convection in the Earth's mantle, thermal convection in clouds, etc. Gasser and Kazimi(1976) have investigated the internal heat generation problem during the onset of convection in a porous medium. Rudraiah and Shekar (1991) have studied convection in a ferrofluid layer in association with a uniform internal heat source, whereas the effect of internal heat generation on the onset of Brinkman-Benard ferro-convection through a porous medium has been analysed by Nanjundappa and Prakash (2010). Sharma et al. (2006) and Kumar et al. (2014, 2015, 2016) thoroughly investigated the thermal and thermosolutal convective problems associated with ferrofluid under rotation, couple-stress and compression along with the simultaneous effects of magnetic field, variable gravity, suspended particles and heat source strength through a Darcy-Brinkman porous medium.

Understanding fluid flow through porous media is of great importance in geophysical fluid dynamics and the recovery of crude oil from the Earth's interior (Kumar et al., 2025). This knowledge is also useful for solidification, filtration equipment and various industries such as the chemical industry, the petroleum industry, etc. Nield and Bejan2017) carried out a detailed investigation about the convection through a porous medium. The Brinkman model provides better results than the Darcy model for high-porosity flow by ensuring more thermal stability to the system. The governing hydrodynamic equations are solved by using conventional perturbation methodologies. The aim of this analysis is to theoretically investigate the stratified ferrofluid stability in the presence of a uniform vertical magnetic field strength and an internal heat source through a Darcy-Brinkman porous medium.

2. MATHEMATICAL FORMULATION

A ferromagnetic fluid bounded by two infinite horizontal layers is considered through a Darcy-Brinkman porous medium under the influences of variable density ρ , kinematic viscosity υ, Darcy-Brinkman effective viscosity $\tilde{\mu}_{ef}$, Darcy-Brinkman medium porosity $\in$, Darcy-Brinkman medium permeability k_1 and thermometric conductivity k . A uniform vertical magnetic field $\mathbf{H}(0,0,H)$ and uniform heat source $Q(z)$ permeate the system, with gravity acting vertically downwards, flowing in a porous medium governed by the Darcy-Brinkman model. A geometrical diagram of the physical problem is shown in Fig. 1.

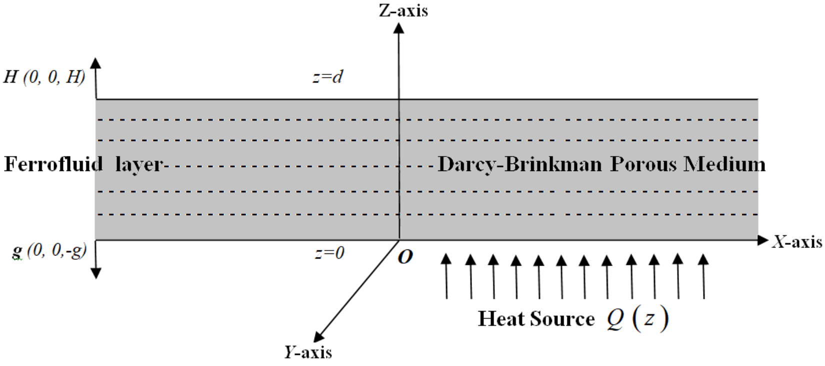

Fig. 1. Geometric diagram of the considered problem

The Continuity of the porous (Darcy-Brinkman) medium having density *ρ* and co-efficient of viscosity *μ*), as depicted in Fig. 1, is governed by the following equations of motion:

$$\frac{\rho}{\in}\left[\frac{\partial \mathbf{q}}{\partial t}+\frac{1}{\in}(\mathbf{q}.\nabla)\mathbf{q}\right]=-\nabla p+\rho \mathbf{X}_{\mathbf{i}}+\mu_0'(\mathbf{M}.\nabla)\mathbf{H}-\frac{\mu}{k_1}\mathbf{q}+\left(\frac{\tilde{\mu}_{ef}}{\in}\right)\nabla^2\mathbf{q}+\frac{\mu_e}{4\pi}\left[(\nabla\times\mathbf{H})\times\mathbf{H}\right] \tag{1}$$

$$\nabla.\mathbf{q}=0 \tag{2}$$

$$\frac{\partial \rho}{\partial t}+\frac{1}{\in}(\mathbf{q}.\nabla)\rho=0 \tag{3}$$

where $\mathbf{X_i}=-\mathbf{g}\lambda_i$ is the external force due to gravity, $\mathbf{q}$ is the velocity of fluid particles, ∇p is the pressure gradient for ferromagnetic fluid, μ_e is the magnetic permeability of medium, μ_0' is the magnetic permeability of vacuum, $\tilde{\mu}_{ef}$ is the Darcy-Brinkman effective viscosity, $\mathbf{H}$ is the magnetic field intensity and $\mathbf{B}$ is the magnetic induction.

The equation for the temperature field, at temperature *T*, with heat source strength $Q(z)$ obeying Fourier's law of heat conduction is as:

$$\left[\in \rho c_v + \rho_s c_s (1-\in)\right]\frac{\partial T}{\partial t} + \rho c_v (\mathbf{q}.\nabla)T = k_T \nabla^2 T + Q(z) \tag{4}$$

where ρ_s is the density of solid material, c_s is the heat capacity of solid material, c_v is the specific heat at constant volume and k is the thermal conductivity of fluid particles.

Maxwell's equations of electromagnetism (with electrical resistivity $\eta = 0$) are:

$$\in \frac{\partial \mathbf{H}}{\partial t} = \nabla \times (\mathbf{q} \times \mathbf{H}) \tag{5}$$

$$\nabla.\mathbf{H} = 0 \tag{6}$$

The equation of state for the density is as:

$$\rho = \rho_0 \left[1 + \alpha (T_0 - T)\right] \tag{7}$$

where, α is the thermal expansion coefficient, ρ_0 is the initial density and T_0 is the temperature at the lower boundary.

For a non-conducting fluid having zero displacement currents, Maxwell's equations become:

$$\nabla.\mathbf{B} = 0 \tag{8}$$

$$\nabla \times \mathbf{H} = 0 \tag{9}$$

where, the magnetic induction *B*, magnetisation *M* and the intensity of magnetic field *H* are connected by the relation of the form (Nield & Bejan, 2006):

$$\mathbf{B} = \mu_0{}'(\mathbf{H} + \mathbf{M}) \tag{10}$$

Now, considering that *M* does not depend on *H* and is a function of *T* only, we can have a relation of the form:

$$\mathbf{M} = M_0[1 + \chi (T_0 - T)] \tag{11}$$

where, T_0 and M_0 are respectively the reference temperature and reference magnetization along with $H = |\mathbf{H}|,\ M = |\mathbf{M}|$, $M_0 = M(T_0)$.

and the pyromagnetic co-efficient

$$\chi = -\frac{1}{M_0}\left(\frac{\partial M}{\partial T}\right)_{H_0} \tag{12}$$

The fundamental state of the system can be described as:

$$\begin{aligned}&\mathbf{q_b} = \mathbf{q_b}(0,0,0),\ p = p_b(z) = p_0 - g\rho_0\left[1+\frac{\alpha\beta' z^2}{2}\right],\ T_b(z) = T_0 - \beta z + Q(z),\\&\mathbf{H} = \mathbf{H_b}(0,0,\mathbf{H_z}),\ \mathbf{M} = \mathbf{M_b}(z),\ \rho = \rho_b(z) = \rho_0(1+\alpha\beta' z).\end{aligned} \tag{13}$$

Let the basic state as described by Eq. (13) be slightly perturbed by giving perturbations of the form:

$$\begin{aligned}&\mathbf{q} = \mathbf{q_b} + \delta\mathbf{q},\ p = p_b(z) + \delta p,\ T = T_b(z) + \theta, \mathbf{H} = \mathbf{H_b} + \mathbf{h}(h_x, h_y, h_z),\\&\rho = \rho_b(z) + \delta\rho,\ \ \mathbf{M} = \mathbf{M_b}(z) + \mathbf{m}(m_x, m_y, m_z).\end{aligned} \tag{14}$$

Here $\delta\rho$ and m respectively represent the changes in density and magnetisation caused by perturbations θ and χ in temperature and concentration, such that

$$\delta\rho = -\alpha\rho_m\theta,\ \ m = -\chi M_0\theta \tag{15}$$

Employing linear stability theory and considering a very small perturbation, the relevant linearised equations (*i.e.,* ignoring nonlinear terms) for perturbation in the magnetised ferrofluid can be defined as:

$$\frac{\rho}{\in}\frac{\partial \mathbf{q}}{\partial t} = -\nabla(\delta p) - g(\delta\rho)\lambda_i - \mu_0'\chi M_0\nabla\mathbf{H}\theta + \mu_0'(\mathbf{M}.\nabla)\mathbf{h} - \frac{\mu}{k_1}\mathbf{q} + \left(\frac{\tilde{\mu}_{ef}}{\in}\right)\nabla^2\mathbf{q} + \frac{\mu_e}{4\pi}\left[(\nabla\times\mathbf{h})\times\mathbf{H}\right] \tag{16}$$

$$\nabla.\mathbf{q} = 0 \tag{17}$$

$$\in\frac{\partial}{\partial t}(\delta\rho) = -\mathbf{w}(D\rho) \tag{18}$$

$$E\frac{\partial\theta}{\partial t} = -\frac{\partial T_b}{\partial z}\mathbf{w} + \kappa\nabla^2\theta \tag{19}$$

$$\in \frac{\partial \mathbf{h}}{\partial t} = (\nabla.\mathbf{H})\mathbf{q} \tag{20}$$

$$\nabla.\mathbf{h} = 0 \tag{21}$$

Where

$$E = \in + (1 - \in)\left(\frac{\rho_s C_s}{\rho C_v}\right),\ \upsilon = \frac{\mu}{\rho},\ \kappa = \frac{k_T}{\rho C_v},\ \lambda_i = (0,0,1),\ D = \frac{d}{dz}.$$

Now, using the method of normal mode, let us consider an exponential solution with a dependence on *x, y* and *t* of the form:

$$\exp\left(ik_x x + ik_y y + nt\right) \tag{22}$$

where, k_x and k_y represents the wave numbers along *x* and *y* directions respectively such that $k = \sqrt{k_x{}^2 + k_y{}^2}$ is the resultant horizontal wave number and *n*, which is in general a complex quantity, represents the growth rate of the harmonic disturbance.

Eqs. (16) - (21) yield

$$\frac{\rho}{\in} n\mathbf{u} = -ik_x(\delta p) + \mu_0{}'\mathbf{M}(D\mathbf{h_x}) - \frac{\mu}{k_1}\mathbf{u} + \left(\frac{\tilde{\mu}_{ef}}{\in}\right)(D^2 - k^2)\mathbf{u} + \frac{\mu_e \mathbf{H}}{4\pi}\left[(D\mathbf{h_x} - ik_x\mathbf{h_z})\right] \tag{23}$$

$$\frac{\rho}{\in} n\mathbf{v} = -ik_y(\delta p) + \mu_0{}'\mathbf{M}(D\mathbf{h_y}) - \frac{\mu}{k_1}\mathbf{v} + \left(\frac{\tilde{\mu}_{ef}}{\in}\right)(D^2 - k^2)\mathbf{v} + \frac{\mu_e \mathbf{H}}{4\pi}\left[(D\mathbf{h_y} - ik_y\mathbf{h_z})\right] \tag{24}$$

$$\frac{\rho}{\in} n\mathbf{w} = -D(\delta p) - g(\delta\rho)\lambda_i + \mu_0{}'\mathbf{M}(D\mathbf{h_z}) - \mu_0{}'\chi M_0(D\mathbf{H})\theta - \frac{\mu}{k_1}\mathbf{w} + \left(\frac{\tilde{\mu}_{ef}}{\in}\right)(D^2 - k^2)\mathbf{w} \tag{25}$$

$$ik_x\mathbf{u} + ik_y\mathbf{v} + D\mathbf{w} = 0 \tag{26}$$

$$n \in (\delta\rho) = -\mathbf{w}(D\rho) \tag{27}$$

$$\left(nE - \kappa D^2\right)\theta = \beta Q'(z) w \tag{28}$$

$$n \in \mathbf{h_x} = \mathbf{H}(D\mathbf{u}) \tag{29}$$

$$n \in \mathbf{h_y} = \mathbf{H}(D\mathbf{v}) \tag{30}$$

$$n \in \mathbf{h_z} = \mathbf{H}(D\mathbf{w}) \tag{31}$$

$$ik_x \mathbf{h_x} + ik_y \mathbf{h_y} + D\mathbf{h_z} = 0 \tag{32}$$

where,

$$\beta = \left| \frac{dT}{dz} \right| \tag{33}$$

and $Q'(z)$ denotes the perturbation in the heat source. Multiplying Eq. (23) by ik_x and Eq. (24) by ik_y and adding, an expression is obtained as

$$\frac{\rho}{\in} n D\mathbf{w} = -k^2(\delta p) + \mu_0' \mathbf{M}(D^2 \mathbf{h_z}) - \frac{\mu}{k_1} D\mathbf{w} + \left(\frac{\tilde{\mu}_{ef}}{\in}\right)(D^2 - k^2) D\mathbf{w} + \frac{\mu_e \mathbf{H}}{4\pi}\left[(D^2 - k^2)\mathbf{h_z}\right] \tag{34}$$

Now, multiplying Eq. (25) by k^2 and subtracting from Eq. (34) after using Eqs. (26) - (33), we can have an expression of the form:

$$\frac{n}{\in}\left[D(\rho D\mathbf{w}) - \rho k^2 \mathbf{w}\right] + \frac{gk^2(D\rho)\mathbf{w}}{\in n} + \frac{\mu}{k_1}(D^2 - k^2)\mathbf{w} - \left(\frac{\tilde{\mu}_{ef}}{\in}\right)(D^2 - k^2)^2 \mathbf{w}$$
$$- \frac{\mu_e \mathbf{H}^2}{4\pi \in n}\left[(D^2 - k^2) D^2 \mathbf{w}\right] + \frac{\mu_0' \chi \mathbf{M}_0 k^2 (D\mathbf{H}) \beta Q'(z) \mathbf{w}}{\left\{nE - \kappa D^2\right\}} - \frac{\mu_0' \mathbf{MH}}{\in n}\left[(D^2 - k^2) D^2 \mathbf{w}\right] = 0 \tag{35}$$

Eq. (35) represents a dispersion relation in terms of the magnetic field and heat source parameter, for a stratified ferromagnetic fluid saturating a Darcy-Brinkman porous medium.

3. EXPONENTIALLY VARYING STRATIFICATIONS

Taking the stratifications in various embedded physical parameters of the form:

$$\left[\rho, \mu, \tilde{\mu}_{ef}, \in, k_1, \mathbf{H}^2, E, \kappa\right] = \left[\rho_0, \mu_0, \tilde{\mu}_{0_{ef}}, \in_0, k_{10}, {H_0}^2, E_0, \kappa_0\right] e^{\beta z} \tag{36}$$

where, $\rho_0, \mu_0, \mu_{0_{ef}}, \in_0, k_{10}, H_0, E_0, \kappa_0$ and β all are constants.

Use of Eq. (36) in Eq. (35) leads to

$$\left[\frac{n}{\in_0}+\frac{\upsilon_0}{k_{10}}-\left(\frac{\tilde{\upsilon}_{0_{ef}}}{\in_0}\right)\left(D^2-k^2\right)\right]\left\{nE_0-\kappa_0 D^2\right\}\left(D^2-k^2\right)\mathbf{w}+\frac{g\beta k^2}{\in_0 n}\left\{nE_0-\kappa_0 D^2\right\}\mathbf{w}$$
$$-\left[\frac{V_A{}^2}{\in_0 n}+\frac{\mu_0{}'\mathbf{M}H_0}{\rho_0\in_0 n}\right]\left\{nE_0-\kappa_0 D^2\right\}\left\{\left(D^2-k^2\right)D^2\mathbf{w}\right\}+\frac{\mu_0{}'\chi\mathbf{M}_0 k^2\beta Q'(z)H_0}{\rho_0}\mathbf{w}=0 \quad (37)$$

where, $V_A{}^2=\frac{\mu_e H_0{}^2}{4\pi\rho_0}$ is the square of Alfvén velocity named after H. Alfvén and $\tilde{\upsilon}_{0_{ef}}$ is the Darcy-Brinkman effective kinematic viscosity.

The boundary conditions (for the case of free boundaries) are defined as

$$\mathbf{w}=D^2\mathbf{w}=0 \quad (38)$$

at z = 0 and z = d.

Now, a proper solution for vertical fluid velocity w satisfying the boundary condition (38) is

$$\mathbf{w}=w_0\sin l\pi z \quad (39)$$

where, w_0 and l are constants and $l=1$ for the lowest mode.

On using solution (39), Eq. (37) yields

$$\left[\frac{n}{\in_0}+\frac{\upsilon_0}{k_{10}}+\tilde{\upsilon}_{0_{ef}}\left(\pi^2 l^2+k^2\right)\right]\left\{nE_0+\kappa_0\pi^2 l^2\right\}\left(\pi^2 l^2+k^2\right)-\frac{g\beta Q'(z)k^2}{\in_0 n}\left\{nE_0+\kappa_0\pi^2 l^2\right\}$$
$$+\left[\frac{V_A{}^2}{\in_0 n}+\frac{\mu_0{}'\mathbf{M}H_0}{\rho_0\in_0 n}\right]\left\{nE_0+\kappa_0\pi^2 l^2\right\}\left(\pi^2 l^2+k^2\right)\pi^2 l^2-\frac{\mu_0{}'\chi\mathbf{M}_0 k^2\beta Q'(z)H_0}{\rho_0}=0 \quad (40)$$

On simplifying Eq. (40), a fourth-degree polynomial is obtained as

$$a_0 n^4+a_1 n^3+a_2 n^2+a_3 n+a_4=0 \quad (41)$$

where, the constant coefficients a_0, a_1, a_2, a_3 and a_4 are defined as

$$a_0=\rho_0 E_0 S_1$$
$$a_1=\left[\left(\rho_0+\tilde{\upsilon}_{0_{ef}}\rho_0 S_1\right)E_0 S_1+\frac{\rho_0\upsilon_0\in_0 E_0 S_1}{k_{10}}+\rho_0\kappa_0\pi^2 l^2 S_1\right]$$

$$a_2 = \begin{bmatrix} \left(E_0 + \kappa_0 \pi^2 l^2\right) \dfrac{\rho_0 \upsilon_0 \in_0 S_1}{k_{10}} + \left(\rho_0 + \tilde{\upsilon}_{0_{ef}} \rho_0 S_1\right) \kappa_0 \pi^2 l^2 S_1 + \left(V_A^{\ 2} \rho_0 + \mu_0' M H_0\right) \\ E_0 \pi^2 l^2 S_1 + \beta Q'(z) k^2 \left(g \rho_0 E_0 - \mu_0' \chi M_0 \beta Q'(z) H_0 \in_0\right) + \tilde{\upsilon}_{0_{ef}} \rho_0 E_0 S_1^{\ 2} \end{bmatrix}$$

$$a_3 = \begin{bmatrix} \dfrac{\rho_0 \upsilon_0 \in_0 \kappa_0 \pi^2 l^2 S_1}{k_{10}} + \left[\left(V_A^{\ 2} \rho_0 + \mu_0' M H_0 + \tilde{\upsilon}_{0_{ef}} \rho_0 E_0 S_1 \kappa_0\right) \pi^2 l^2 S_1 - g \beta Q'(z) k^2 \rho_0 \right] \\ \left(E_0 + \kappa_0 \pi^2 l^2\right) - \mu_0' \chi M_0 k^2 \beta Q'(z) H_0 \in_0 \end{bmatrix}$$

$$a_4 = \left[\left(V_A^{\ 2} \rho_0 + \mu_0' M H_0\right) \kappa_0 \pi^4 l^4 S_1 - g \beta Q'(z) k^2 \rho_0 \kappa_0 \pi^2 l^2 \right]$$

where, $S_1 = \left(\pi^2 l^2 + k^2\right)$.

Eq. (41) is biquadratic in the growth rate and therefore must give 4roots.

4. DISCUSSION AND RESULTS

The causalities of different embedded parameters accountable for the stability/instability of the system are discussed elaborately to have an idea about the convectional onset in the ferrofluid layer through a Darcy-Brinkman porous medium.

A. Stable stratification cases:

Case1: Let $\alpha_1, \alpha_2, \alpha_3$ and α_4 be the 4 roots of Eq. (41). Now, with $\beta < 0,\ \mu_0' > 0$ and $g \rho_0 E_0 < \mu_0' \chi M_0 \beta Q'(z) H_0 \in_0$, a_0, a_1, a_2, a_3 and a_4 will be ≥ 0 and therefore Eq. (41) does not possesses any +ve root of *n*. So, $\alpha_1 \alpha_2 \alpha_3 \alpha_4 = \left(\dfrac{a_4}{a_1}\right) > 0.$ This is clearly suggesting the stability of the system for disturbances with any wave numbers.

Case2: If $H_0 = 0,\ V_A^{\ 2} = 0$ (*i.e.*, in the absence of magnetic field), under the conditions $\beta < 0$ and $\mu_0' > 0,$ the system may have the effect of stabilization.

B. Unstable stratification cases:

Case1: If $\beta > 0,\ \mu_0' > 0,\ g \rho_0 E_0 > \mu_0' \chi M_0 \beta Q'(z) H_0 \in_0$ and $\left(V_A^{\ 2} \rho_0 + \mu_0' M H_0\right) \pi^2 l^2 S_1 > g \beta Q'(z) k^2 \rho_0$ then a_0, a_1, a_2, a_3 and $a_4 \geq 0$ and therefore Eq. (41) does not have any +ve root. So, the system possesses the effect of stabilization for disturbances with any wave numbers.

Case2: If $\beta > 0$ and $\left(V_A{}^2\rho_0 + \mu_0{}'MH_0\right)\pi^2 l^2 S_1 < g\beta Q'(z)k^2\rho_0$ then a_4 will be < 0 and therefore Eq. (40) has at least one +ve root, thereby implying the instability of the system.

Case3: If $H_0 = 0, V_A{}^2 = 0$ (*i.e.*, in the absence of a magnetic field), a_4 will be < 0 for $\beta > 0$. Hence, the system will be unstable for any value of the wave numbers.

In unstable stratification, the variation in the growth rate parameter n, for various parameters like effective Darcy-Brinkman kinematic viscosity $\upsilon_{0_{ef}}$, Darcy-Brinkman porosity $\left(\in_0\right)$ of the medium, Darcy-Brinkman permeability $\left(k_{10}\right)$ of the medium, Alfven velocity square $\left(V_A{}^2\right)$ and heat source parameter $Q'(z)$ has been analytically examined with the help of following derivative equations *i.e.*, $\frac{dn}{d\upsilon_{0ef}}, \frac{dn}{d\in_0}, \frac{dn}{dk_{10}}, \frac{dn}{dV_A{}^2}, \frac{dn}{dQ'(z)}$ respectively.

$$\frac{dn}{d\upsilon_{0_{ef}}} = -\frac{\frac{\rho_0 \in_0 S_1 n}{k_{10}}\left\{E_0 n^2 + \left(E_0 + \kappa_0\pi^2 l^2\right)n + \kappa_0\pi^2 l^2\right\}}{4a_0 n^3 + 3a_1 n^2 + 2a_2 n + a_3} \tag{42}$$

$$\frac{dn}{d\in_0} = -\frac{\left[\begin{array}{l}\frac{\rho_0\upsilon_0 E_0 S_1 n^3}{k_{10}} + \left\{\left(E_0 + \kappa_0\pi^2 l^2\right)\frac{\rho_0\upsilon_0 S_1}{k_{10}} - \beta Q'(z)k^2\mu_0{}'\chi M_0 H_0\right\}n^2 \\ + \left\{\frac{\rho_0\upsilon_0\kappa_0\pi^2 l^2 S_1}{k_{10}} - \beta Q'(z)k^2\mu_0{}'\chi M_0 H_0\right\}n\end{array}\right]}{4a_0 n^3 + 3a_1 n^2 + 2a_2 n + a_3} \tag{43}$$

$$\frac{dn}{dk_{10}} = \frac{\frac{\rho_0\upsilon_0 \in_0 S_1 n}{k_{10}{}^2}\left\{E_0 n^2 + \left(E_0 + \kappa_0\pi^2 l^2\right)n + \kappa_0\pi^2 l^2\right\}}{4a_0 n^3 + 3a_1 n^2 + 2a_2 n + a_3} \tag{44}$$

$$\frac{dn}{dV_A{}^2} = -\frac{\rho_0\pi^2 l^2 S_1\left\{E_0 n^2 + \left(E_0 + \kappa_0\pi^2 l^2\right)n + \kappa_0\pi^2 l^2\right\}}{4a_0 n^3 + 3a_1 n^2 + 2a_2 n + a_3} \tag{45}$$

$$\frac{dn}{dQ'(z)} = \frac{\frac{\rho_0\upsilon_0 E_0 S_1 n^3}{k_{10}} + \left(E_0 + \kappa_0\pi^2 l^2\right)\frac{\rho_0\upsilon_0 S_1}{k_{10}}n^2 + \left\{E_0 n^2 + \left(E_0 + \kappa_0\pi^2 l^2\right)n + \kappa_0\pi^2 l^2\right\}}{4a_0 n^3 + 3a_1 n^2 + 2a_2 n + a_3} \tag{46}$$

Derivatives (42)–(46) show that the growth rate increases with increasing medium permeability and heat source parameters. This means that the medium permeability and heat source parameters have an effect of destabilisation on the system. The growth rate decreases with increasing effective kinematic viscosity and the square of the Alfvén velocity, suggesting an effect of stabilisation from these parameters. The average implications of Darcy-Brinkman have a double property because the growth rate increases and decreases under certain conditions (Kumar et al., 2025).

5. CONCLUSION

A linear instability/stability analysis followed by the normal mode method is taken into account to discuss the effect of convection onset in the ferrofluid layer through a Darcy-Brinkman porous medium, and a dispersion relation is obtained in terms of thermal and solute Rayleigh numbers. Further, the case of a stationary convection ferrofluid layer is also discussed, and a relationship between thermal and solute Rayleigh numbers is obtained to study the effect of various embedded parameters. The critical thermal and solute Rayleigh numbers can be obtained with the help of the critical dimensionless wave number for varying values of physical parameters.

DISCLAIMER (ARTIFICIAL INTELLIGENCE)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of this manuscript.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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