Mathematical Modeling of Effective Distribution System for Resources and Facilities in Disaster Management by using Optimization Techniques

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Abstract

Emergency resources allocation is essential to the emergency rescue effectiveness, and it has become a research topic for emergency rescue. Emergency resource is important for people when any disaster occurs. Considering, dynamic demand, emergency resource allocation schedule is more challenging. The main problem is to determine the optimal stock of emergency resource for a supplier centers to improve the relief efforts. This paper studies the dynamic demand and which is defined as a set. Then an emergency resource allocation model with some uncertain data is presented. Finally, a case study is taken to show the actual working of the presented model.

Keywords- Emergency resource, Dynamic demand, Risk value, entropy, Cluster Centers, Demand weight,

Introduction-

Recently, the main focus of disaster management system is based on effective resource planning. Managing response to natural, man-made, and technical disasters is becoming increasingly important in the light of climate change, globalization, urbanization, and growing conflicts. Sudden onset disasters are typically characterized by high stakes, time pressure, and uncertain, conflicting or lacking information. Since the planning and management of response is a complex task, decision makers of aid organizations can thus benefit from decision support methods and tools. A key task is the joint allocation of rescue units and the scheduling of incidents under different conditions of collaboration. The qualitative framework and formalized risk assessment are needed in uncertainty risk analysis. In such case, a set is introduced to describe the dynamic demand. The related research can be divided into pre-disaster operations (facility location-routing, stock) and post-disaster operations (relief distribution and transportation). The pre-disaster operation is high related with response decisions. Therefore, preparedness and response planning are necessary.

Research Methodology-

The methodology about emergency allocation includes dynamic demand analysis and robust optimization. The dynamic demand of emergency resource is analyzed in the first stage, to obtain the quantity and distribution of dynamic demand. Then, a robust optimization model is introduced to determine the optimal stock of emergency resource in the second stage.

Dynamic demand analysis-

Due to the uncertain disasters, it is almost impossible to know the exact time and severity of an accident. It is very difficult to estimate the emergency resource in demand exactly in advance .However, combining the relief experience and forecasting approach, the fluctuation range of demand is available. In this study the dynamic demand can be expressed as a set ($d^0_{mi}-d_{mi}$, $d^0_{mi}+d_{mi}$). Here, d^0_{mi} refers to the nominal demand of dynamic demand, namely the average demand. The parameter d_{mi} refers to the disturbance of dynamic demand, belongs to uncertain parameter. The distribution of resources appears to be dispersing. It's infeasible to research the relief demand of each disaster location. Therefore, we could divide the disaster region into several disaster areas. At last, the demand quantity of them is calculated. The procedure can be illustrated in Fig 1.

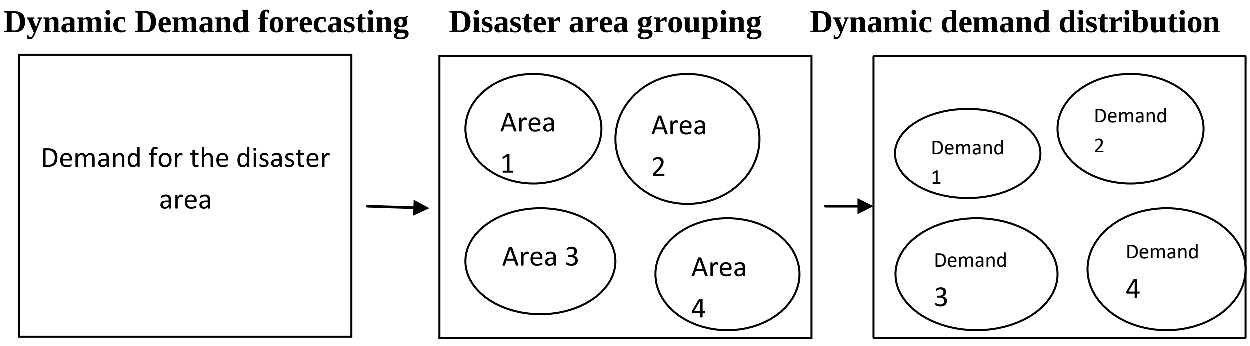

Fig.1 (Partition of disaster region into disaster areas)

Dynamic demand forecasting-

In terms of the demand amount of emergency resource, the transportation ministry of India has issued some documents about the general requirement for the disaster area. According to the relevant regulations, the quantity and categories of emergency resource for a given disaster area can be obtained by the Eq. (1).

$$D_m = Q_m \times L_t \; m \in M \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (Eq. 1)$$

Where, *m* denotes the type of emergency resource; D_m denotes the dynamic demand quantity of the type k emergency resource for the disaster area; Q_m denotes the general requirement according to the relevant document; *L* denotes the comprehensive risk adjustment coefficient of the disaster area in certain time interval. Taking the commodity distribution for example, the requirement for the terminal in disaster area is shown in Table 1.

Table 1 The stock requirement of a commodity-

Terminal Scale(Units)	**0.1-0.5**	**0.5-1**	**1-5**	**5-10**	**10-15**	**15-30**	**More than 30**
Main terminal Capacity (Units)	1	1.5	2	4	5.5	7.5	9
Other terminal capacity (Units)	0.2	0.3	0.4	0.8	1.1	1.5	1.8

Table-2 The criteria of risk value assessment-

Number of indicator	**Indicator of risk assessment (R_θ)**	**Threshold of indicator K_θ**
1	Average disaster per year	280000
2	The no. of serious disaster per year	16
3	The no. of casualties per year	600
4	Throughput of commodities per year	80000

5	Throughput of persons per year	10000
6	Throughput of other factors per year	12000

$$R = \sum_{\theta=1}^{6} \left(\frac{R_\theta}{K_\theta}\right) \quad \text{………………………………..(Eq. 2)}$$

Where, θ is the index of risk indicator, R_θ is the θth risk indicator, K_θ is the threshold of the θth risk indicator, which refers to the criterion risk level with the disaster area is 50000m^2.

Disaster area grouping-

According to the spatial distribution of emergencies at disaster area, we aim to divide the study region into several disaster areas, namely disaster area grouping. The disaster area is represented by its centre and scope. In this section, the centre and the scope of each disaster area should be determined. K-means clustering is a typical spatial clustering method.

Dynamic demand distribution-

After disaster area grouping, we should determine the demand quantity of these disaster areas. This section develops the demand distribution method of emergency resource. The total demand of the whole disaster region is given. Therefore, the key problem is to identify the demand proportion of disaster area occupied in the whole disaster region, named demand weight. The following summarizes the main steps of dynamic demand distribution.

Step 1: Evaluation indicators for demand distribution

Suppose there are k disaster areas in the study region. Considering the actual situation in maritime emergency logistics, three indicators are proposed to determine the demand distribution, the number of maritime accidents, the number of casualties, and the number of damaged ships. The demand assessment matrix A is formatted by:

$$\mathbf{A} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{k1} & a_{k2} & a_{k3} \end{bmatrix} \quad \text{…………………….(Eq. 3)}$$

Where, the element $r_{ij,}$ in which i = 1.2.k, j = 1.2.3 is the original value of the jth evaluation indicator in disaster area i. Considering the different measurement scales associated with these criteria, the original value should be standardized by Eq (4).

$$p_{ij} = r_{ij} / \sum r_{ij} \quad j=1,2,3 \text{……………………………...…(Eq. 4)}$$

p_{ij} represents the standardized evaluation value, then the standardized matrix can be obtained by Eq (5)

$$\mathbf{B} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{k1} & p_{k2} & p_{k3} \end{bmatrix} \quad \text{.....................(Eq. 5)}$$

Step 2: Calculating the entropy value of evaluation indicator:

The entropy value of the jth evaluation indicator in demand distribution is given as:

$$e_j = -\ln(k)^{-1}\sum p_{ij} \ln p_{ij} \quad \text{..................................(Eq. 6)}$$

Step 3: Calculating the entropy weight of evaluation indicator

The entropy weight of the *j*th evaluation indicator in demand distribution is following.

$$E_j = \frac{1-e_j}{\sum_{j=1}^{3} 1-e_j} \quad \text{...(Eq. 7)}$$

Step 4: Demand proportion identification of the disaster area

Sum up all the evaluation indexes for one disaster area with weight sum method, the demand weight of emergency resource for disaster area *i* is obtained.

$$w_i = \sum_{j=1}^{3} E_j p_{ij} \text{...(Eq. 8)}$$

The greater of the weight for emergency resource demand, the greater share of the emergency resource demand in the area.

Step 5: The nominal dynamic demand

The nominal demand of type *m* emergency resource in disaster area *i* is derived by the following equation.

$$d^0_{mi} = D_m w_i \text{ ....(Eq. 9)}$$

Robust optimization model-

Problem definition-

For the emergency resource allocation under dynamic demand, the emergency resource demand of each disaster area is uncertain, varying in a set. This paper aims to determine the optimal quantity of emergency resource in each supplier, achieving the optimal of economic and effectiveness. A general overview of maritime emergency resource allocation can be described in Fig. 2

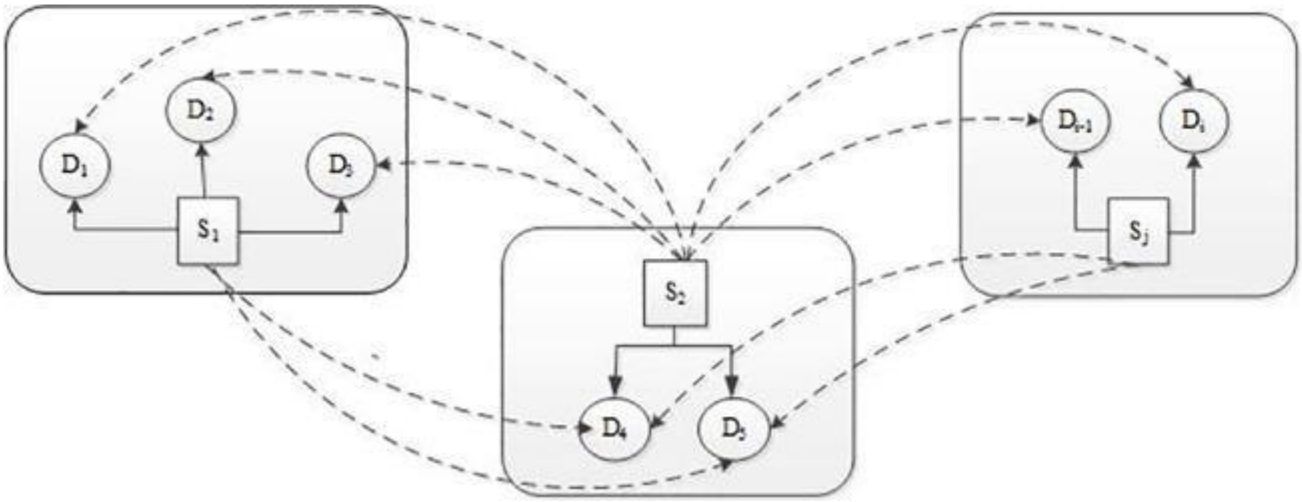

Fig.2 (Problem definition chart) Emergency resource allocation-

As is depicted in Fig. 2 *S* is the emergency resource supplier center, *D* is the disaster area, the *i*th disaster area can be denoted by D_i, and the jth supplier can be denoted by S_j. Once any disaster occurs, the emergency resource should be transit from the supplier center as near as possible. For example, if the demand in disaster area D_1 generates, the emergency resource should be transit from the supplier center S_1, if the emergency resource is insufficient, the supplier center S_2 will be the candidate. Supposing there are altogether *I* disaster areas, *J* emergency resource supplier centers and M ($m \in M$) types emergency resource. To simplify the model, main assumptions of the proposed model are as follows:

1. Any supplier center can provide emergency resources to any disaster area. The maritime rescue is relatively difficult and time consuming. So, in practice, the transportation time is unlimited.

2. The emergency resources in one supplier center cannot be transferred to another. Usually, the resource supplier center is far away from another. The following variables are defined for the model:

Vj : the maximum storage capacity of supplier S_j.

t_{mij} : the transportation time for the type *m* emergency resource from supplier S_j to demand point D_i. α : the penalty coefficient of demand, it is noteworthy that the range for it is more than 1.

d_{mi} : the dynamic demand for the type m emergency resource in disaster area D_i. It takes value in the interval ($d^0_{mi} - d_{mi}$, $d^0_{mi} + d_{mi}$)), with mean equals to the nominal value d^0_{mi}.

x_{mj} : the quantity of type *m* emergency resource reserved in supplier S_j. y_{mij} : the quantity of type *m* emergency resource assigned from supplier S_j to demand point D_i. z_{mj} : the compensation quantity of type *m* emergency resource for supplier S_j When the emergency resource is insufficient.

Model-

The optimal quantity of emergency resource will be determined by the model, aims to achieve the optimal of total time and loss. Total time of emergency resource allocation refers to the total time of all emergency resource transportation from the supplier center to the disaster area. Given the above assumptions and notations, the maritime emergency resource allocation model can be stated as follows:

$$f_1 = \min \sum_{m\in M}\cdot \sum_{i\in I}\cdot \sum_{j\in J}\cdot t_{mi} y_{mij}$$

$$f_2 = \sum_{m\in M}\cdot \{\sum_{i\in I}\cdot \sum_{j\in J}\cdot(\sum_{i\in I} z_{mi}\}$$

$$\text{s.t.} \begin{cases} \sum_{m\in M} x_{mj} \le V_j \ \forall j \\ \sum_{i\in I} y_{mij} \le x_{mj} \ \forall m, j \\ \sum_{i\in I} y_{mij} + z_{mi} \ge d_{mi} \ \forall m, i \\ x_{mj}, y_{mij}, z_{mi} \ge \forall m, i, j \end{cases} \qquad \ldots\ldots\ldots\text{(Eq.10)}$$

The emergency resource allocation model is a multi-objective stochastic programming. The first objective function is to minimize the total transportation time for emergency resource allocation. The second objective function is to minimize the total loss for emergency resource allocation. The first part is the loss due to excess reserve, could be denoted by quantity of idle emergency resource, the second part is the loss due to shortage, and could be denoted by the quantity of compensation emergency resource and the penalty coefficient. The first constraint ensures that the quantity of emergency resource reserved in the supplier center does not exceed its capacity. The second constraint guarantee the quantity of emergency resource assigned from supplier S_j cannot exceed the quantity reserved in it. The third constraint state the demand of disaster area should be satisfied. The last constraint ensures the variables are significant.

Case Study of a disaster occurred place (say X) Data approximation-

Let, there are 476 terminals in place X, area in India(say X), among these, 214 terminals are more than ten thousand units .The terminals distribution of different grade is shown in Table 3. The data is assumed by real situation. For example, as the Table 3 shows, there are 63 commodity terminals; the scale is between 1000 to 5000 units.

Table 3

The terminals in place X-

Terminal Scale (Ten thousands Units)	**0.1-0.5**	**0.5-1**	**1-5**	**5-10**	**10-15**	**15-30**	**More than 30**
Main terminal Capacity (Units)	63	10	16	16	6	2	2
Other terminal capacity (Units)	81	87	94	43	45	6	5

The risk indicator statistics in Place X can be shown in Table 4. As place X is 159,500 square kilometers and the standard sea area is 50,000 square kilometers, the area conversion coefficient is 5 / 15.95 = 0.3135.

Table 4

The risk indicator of place X-

Indicator of risk assessment (R_θ)	**2017**	**2018**	**2019**	**Average Value**	**Standard Value of Indicator**
Average disaster per year	162119	162120	162000	162080	50812
The no. of serious disaster per year	22	28	28	26	8.1
The no. of casualties per year	1260	1741	890	1297	406

Throughput of commodities per year	100000	89000	88500	92500	28998
Throughput of persons per year	2973	2581	2604	2719	853
Throughput of other factors per year	17100	22466	23595	21052	6600

Dynamic demand analysis

Dynamic demand forecasting of place X-

According to table 1 and table 4, the general requirement of a commodity calculated as 421.8 Units. Put the standard value of indicator as shown in Table 4 and the threshold of indicator as shown in Table 2 into the risk value calculation Eq.(2), the risk value of place X is estimated:

$$R=\frac{1}{6}\sum_{i=1}^{6}\left(\frac{R_i}{K_i}\right) = 1.89 \text{……………..(Eq. 11)}$$

According to the Eq. (11), the risk adjustment coefficient can be calculated as R = 1.89 .Thus, the Commodity demand of place X is D = 1.89×421.8 = 797.20 Units approx.

Disaster area grouping-

Based on the minimum circular coverage and spatial clustering method, the disaster areas grouping steps are as follows:

Step 1: Draw circle to cover the disasters points. Based on the minimum circle coverage theory, draw several unequal size circles to cover the accidents points in the map. The number of circle is the cluster number. The center of circle is considered as the initial clustering center. Step 2: Identification of disaster area

- ① Calculate the Euclidean distance between each disaster point *x* to the clustering center, and put the disaster point into the group closest to it.
- ② Update the cluster center.
- ③ Calculate the square error criterion function.
- ④ Repeat step ①-③ until the criterion function get convergence.

In terms of disaster point distribution map in place X, 8 unequal size circles are drawn to cover the disaster points. The circles should be as small as possible, to achieve the largest of the blank area outside the circle and the highest of accident points concentration. 8 circles are formed P_1P_8. The center of circle P_1-P_8 is considered as the initial clustering center.

Table 5

The Disaster Area in Place X-

Disaster Area	The no. of disasters	The no. of casualties	The no. of damaged Houses
P_1	10	49	07
P_2	11	91	12
P_3	13	65	09
P_4	08	82	09
P_5	12	92	12
P_6	11	96	11
P_7	08	54	04
P_8	07	28	01

Dynamic demand distribution-

After disaster area grouping, apply the Entropy theory method, the dynamic demand distribution of place X can be determined. The standard demand assessment matrix A of 8 disaster areas is formatted as Eq. (12)

$$P = (p_{ij})_{m \times k} = \begin{bmatrix} 0.125 & 0.088 & 0.108 \\ 0.138 & 0.163 & 0.185 \\ 0.163 & 0.117 & 0.138 \\ 0.1 & 0.147 & 0.138 \\ 0.15 & 0.147 & 0.138 \\ 0.138 & 0.172 & 0.169 \\ 0.1 & 0.097 & 0.062 \\ 0.088 & 0.050 & 0.015 \end{bmatrix} \quad \text{………………………………(Eq. 12)}$$

By Eq. (6), the entropy value of evaluation indicator can be calculated, the entropy value for the number of maritime accidents, the number of casualties, the number of damaged ships are e_1 = 0.988, e_2 = 0.969, e_3 = 0.935 respectively. By Eq. (7), the entropy weight of evaluation indicator for the number of maritime accidents, the number of casualties, the number of damaged houses are E_1 = 0.111, E_2 = 0.287, E_3 = 0.602 respectively. The demand weight $\boldsymbol{w_i}$ for the 8 disaster areas are obtained by the Eq. (8), as is shown in Table 6. **Table 6**

Demand weight for the 8 disaster areas.

Disaster Area	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
Demand weight w_i	0.104	0.173	0.135	66.8	0.175	0.167	0.076	0.033

The nominal demand d_i of oil dispersant for 8 disaster area is derived by $d_{mi} = D_m w_i$, as shown in Table 6. When the demand disturbance level is 2%, 5%, 10%, the dynamic demand for the 8 disaster area is shown in Table 7.

Table 7 Nominal Demand for disaster areas-

Disaster Area	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
Commodity demand	50.7	84.4	65.8	66.8	85.4	81.4	37.1	16.1

Table 8 Dynamic demand for disaster areas-

Disaster Area		P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
Disturbance 5%	**Upper Limit**	53.2	88.6	69.1	70.1	89.7	85.5	38.9	16.9
	Lower Limit	48.2	80.2	62.5	63.5	81.1	77.3	35.2	15.3
Disturbance	**Upper**	55.8	92.8	72.4	73.5	93.9	89.5	40.8	17.7
10%	**Limit**								
	Lower Limit	45.6	75.9	59.2	60.1	76.9	73.3	33.4	14.5
Disturbance 20%	**Upper Limit**	60.8	101.3	78.9	80.2	102.5	97.7	44.5	19.3
	Lower Limit	40.6	67.5	52.60	53.4	68.3	65.1	29.7	12.9

Let there are 5 emergency resource supplier center located in Place X denoted by S_1, S_2, S_3, S_4, S_5 in the map. In the last section, the disaster areas can be determined, denoted by D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8. Using the position data of disaster areas and supplier center, the distribution map of supplier centers and disaster areas could be drawn on the Approximated map. The maritime emergency supplier centers and disaster areas map of place X is shown in Fig.3

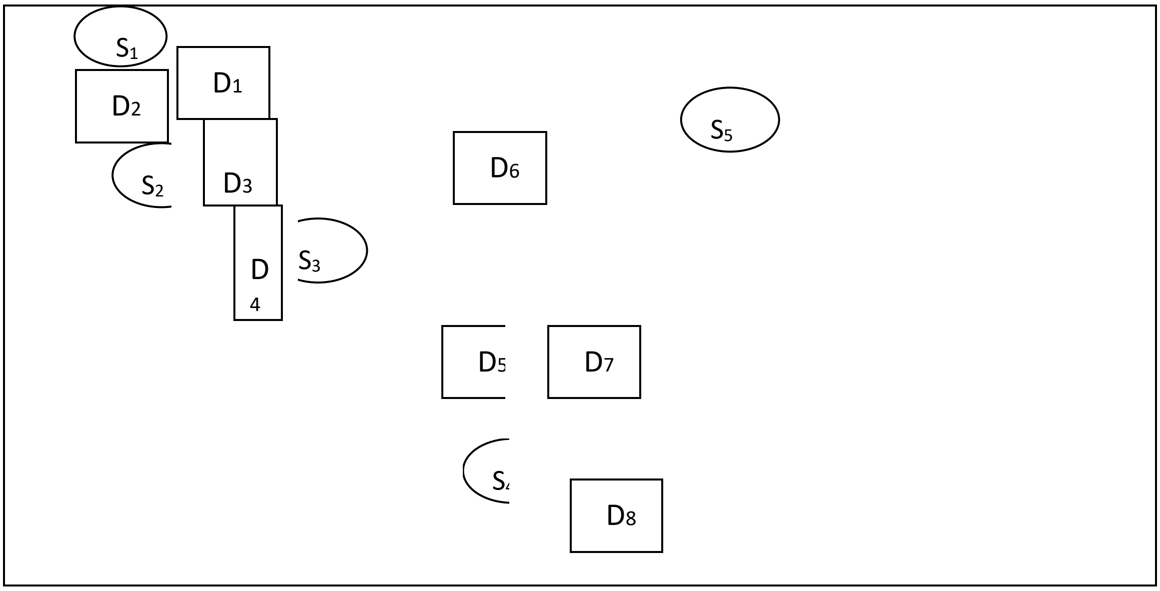

Fig. 3 (Area map of a particular place X) Distribution of suppliers and disaster areas-

The location data of supplier center and disaster area can be assumed as per the map in Fig. 3, by the use of distance measurement tool, the assumed distance between emergency resource supplier center and the disaster area center is shown in Table 9.

Table 9 Distance between supplier center and the disaster area center (kilometers)-

Supplier Center →	S_1	S_2	S_3	S_4	S_5
Disaster Area ↓					
D_1	105	215	353	630	745
D_2	209	72	205	473	562
D_3	322	97	103	386	500
D_4	405	165	40	290	383
D_5	508	255	72	210	295
D_6	702	480	292	50	89
D_7	575	355	177	228	305
D_8	647	430	245	335	400

Table 10

The transportation time (hour)-

Supplier Center →	S_1	S_2	S_3	S_4	S_5
Disaster Area ↓					
D_1	1.62	3.32	5.45	9.72	11.49
D_2	3.22	1.11	3.16	7.29	8.67
D_3	4.97	1.49	1.59	5.95	7.71
D_4	6.25	2.55	0.62	4.47	5.91
D_5	7.84	3.93	1.11	3.24	4.55
D_6	10.83	7.41	4.50	0.77	01.37

D_7	8.87	5.48	2.73	3.52	4.71
D_8	9.98	6.63	3.78	5.17	6.17

Results-The above table shows the practicality of the proposed model for real life situation. The disaster statistics show that, most of the emergencies occur in the range of 10–30 kilometres area, and emergency resources supplier centre is mainly built in the port terminal. Therefore, emergency resources from the supplier centre to the disaster area are important. Let the average speed of a vehicle is 35km per hour, so the average speed of emergency resources from the supplier centre to the disaster area is 64.82 km /h. Table 9, provides the distance from the supplier centre to the accident centre, the transportation time is shown in Table 10.

Conclusion- The presented paper can be very useful in disaster area grouping and determine the demand of a disaster prone area. We can take some realistic data from the disaster affected area and would be able to get some better solution for the problem. This paper also provided us the better allocation plan for the resources helpful in disaster situation. The limitation of the proposed model is that sometime it is not possible to get the appropriate data for a particular place, in such a case the parameters of the model cannot be defined effectively. There is also a limitation of the proposed model when the number of destination are very large then the calculation becomes more tedious.

Future Scope - This research paper can be useful for the future aspects of new research in the field of disaster management. We can take accurate data for a specific region and analyzing the data by using the methodology in this paper we can abstract some new results also .Also by using some simulation algorithm some new mathematical models can developed.

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