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Mathematical Modelling for Teaching Learning Process using the Concept Mapping

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Abstract

In education, it is in general desirable that the educational process is not to cram in knowledge but to build up the knowledge based on learner's understanding of the related concept. Concept teaching mapping have been presented in pedagogical literature, and it can more easily and rapidly represent a part of learner's understanding than the written and oral examination. Though the concept map is a powerful and useful tool for teachers to know learner's understanding, supporting method to create and correct it has never been considered. In this paper, we present a method to support collaborative learning using the concept map based on structural modelling, FISM (Flexible Interpretive Structural Modeling). By using this method, learners are supported to construct the consistent concept map. Teachers can know minimum leading point by comparing his/her concept map with a consensus model of learners. This concept mapping is very useful for micro teaching students to learning programme.

Keywords: Teaching Learning, Matrix, Concept Map, FISM.

1. INTRODUCTION

Now, many researchers and educators regard constructive learning as important (Inaba, 1999). As a related idea, "concept map" to support building up the knowledge based on learner's understanding of related concepts, has been presented in pedagogical literature. It can more easily and rapidly represent part of learner's understanding than the written or oral examination, so teachers have often used it as a method of examination. Though the concept map is a powerful and useful tool for teachers to determine learner's understanding, it has never been considered as a supporting method to create and correct it. It is also difficult to objectively evaluate and compare concept maps created by learners. On the other hand, currently, computer supported collaborative learning has been one of the main topics on educational application of the Internet. In the CSCL system, it is not easy to grasp the state of the learner's understanding because the teacher cannot communicate with the learner directly as in a face-to face talk. We consider that grasping the learner's understanding is one of the most important subjects and the concept map might become useful and powerful tool in such a field too (Tsai, 2001; Saito, 2000).

In this paper, we present the following method to support teachers and learners in collaborative learning using the concept map on structural modelling.

1. Expression and identification of the concept labels with explanatory note;
2. Construction of the consistent concept map;
3. Calculation of the minimum leading point.

By using this method, learners might get some support to construct the consistent concept map, and to combine some concept maps constructed by them into a consensus model. Teachers can know a minimum leading point by comparing his/her concept map with a consensus model among learners. Related work, FIRS (Fuzzy Item Relational Structure Analysis) (Itoh, 1994) supports creating a consensus model for teaching material among teachers using some graphs based on FISM constructed by items of lecture and their relations, and using feedback of examination after the class. In the FIRS, the graphs are only used by teachers to create a consensus to our concept map as a kind of teaching aid.

In the next section, we define the concept map referred to in this paper. In the third section, we present a collaborative learning process. Next, we describe the application of a proposed method as an example. Finally, we conclude and describe future work.

2. MATHEMATICAL CONCEPT OF MAPPING

The concept map, in general, is a two-dimensional diagram that shows some concept labels, which are keywords to show learning content, and their links. The proposition that might be a simple means of linking is drawn on the map. There are two types of concept map, hierarchical or nonhierarchical. The hierarchical concept maps, which higher/lower relation and upper/lower position among the concept labels are given equal meaning, are effective for learning with memory. We especially focus on the hierarchical concept map, which is defined as follows.

Definition 1: set of concept labels

A set of concept label cl^t presented by teacher. m is a number of cl^t.

$$CL^t = \{cl_1^t, cl_2^t, \dots, cl_m^t\} \quad (1)$$

A set of concept label cl^s expressed by learners. N is a number of cl^s.

$$CL^s = \{cl_1^s, cl_2^s, \dots, cl_m^s\} \quad (2)$$

Definition 2: Concept map

Concept map CM^t, which is two-dimensional Euclid space arranged and linked the elements of $CL^t$by teacher.$CM^t$is drawn only concept labels. $CM^s$and $CM^{s'}$ $a$re drawn by learners.

Definition 3: Relation among the concept labels

Binary matrix R^t expresses whether concept labels are linked or unlinked on the concept map. R^s is the same as R^t, where R^s (S = 1, 2, …, N) is a three-dimensional matrix when the number of learners is N. R^s is decided by collaborative learning among the learners.

To be constructed the hierarchical concept map needs to consist of the transitive relationship between the concept labels. So, we propose to introduce the FISM (Flexible Interpretive Structural Modeling) (Ohuchi, 1989) to create the hierarchical Concept map.

FISM is an extended and improved version of ISM (Interpretive Structural Modeling) developed by Warfield (Warfield, 1976). The computer algorithm of FISM is based on the partially filled reachability matrix (PR-matrix) model. The PR-matrix model is an extension of a reachability matrix (R- matrix) model and has great utility in all phases of ISM.

In this paper, we suppose that is always right; therefore must be a partial ordering matrix that is equivalence matrix. Onthe other hand, we suppose the learner has ordinal thinking that might include misunderstandings; therefore CM^sneeds to be a more flexible representation than CM^t. So, R^s is pseuso-ordering matrix that is, reflexive and transitive mtrix and that includes partial order and equivalence relation.

3. COLLABORATIVE LEARNING PROCESS

The following are two methods to create the concept map.

1. Teachers indicates his/her own concept labels to learners, and learners decide the relationship between them.
2. Learners express their concept labels, and teacher and learners decide the relationship by collaborative learning.

Compared to concept maps, method 1 is easier than 2, but we select method 2 to build up the knowledge and change the concepts for learners. In this research, we propose following the collaborative learning process for method 2.

[Learning process of learners]

The learners process of learners consists of five phases.

1. Expression of the concept labels with explanatory notes;
2. Identification of the concept labels among the learners;
3. Decision of relations among the concept labels on a concept map.
4. Making a consensus model among learners; and
5. Rewriting a concept map of learners.

[Learning process of teacher]

The learning process of the teacher also consists of five phases.

i. Indication of the concept labels with explanatory note;
ii. Identification of the concept labels among teacher and learners;
iii. Decision of relations among the concept labels of teacher;
iv. Making an opinion graph to lead;
v. Leading to correct knowledge from the elements of disagreement.

In this paper, we assume the following collaborative learning environment aided by a computer that learners can use.

- Each learner can use a computer.
- Learners can communicate by chat.

In the next section we explain detail of their phases.

3.1 Expression and identification of the concept labels with explanatory notes

Both teacher and learners express the concept labels with explanatory note. They identify the concept labels. If they judge concept labels those are not same notation but similar to expression are the same, then they can identify such labels. Here, we introduce the resemblance based on heuristic that “the documents which include many shared keywords have closely connection” to support identification of the concept labels. In the following, we explain the case that the teacher identifies among the labels of the teacher and learners. This method can also apply to the case to identify the labels among learners.

Here, we define the vector to calculate the resemblance, which is made from the frequency of keywords these are nouns extracted from the explanation of concept labels as a result of using the Japanese morphological analysis system “Chosen”. $cl_i^t(i = 1,2, \dots, m)$is a vector to represent a concept label of teacher, where m is a number of concept labels of teacher.

$$cl_i^t = \{w_{i1}^t, w_{i2}^t, \dots, w_{iX}^t\} \quad (3)$$

where,X is a number of keywords.

$cl_j^s(j = 1,2, \dots, n)$is a vector to represent a concept label of learners, where, n is a number of concept labels of learners.

$$cl_j^s = \left\{w_{j1}^s, w_{j2}^s, \dots, w_{jX}^s\right\} \quad (4)$$

Then S_{ij}represents a resemblance calculated by the inner product of cl_i^t and cl_j^s.

$$S_{ij} = cl_i^t . cl_j^s \quad (5)$$

In the next phase, the teacher identifies the concept labels. Here, we propose two methods to support identification at the same time. One is a method to maximize the sum total of S_{ij}, another one is a method using the multiple-dimensional scaling (MDS).

3.1.1 Decide combinations using the Hungarian method

We introduce the Hungarian method (HM) to decide the combination of concept labels of the teacher and learners. Using the HM, the number of concept labels of the teacher and learners must be equal, so that we adopt a method to make the labels the same using temporary labels, whose resemblance is 0, and which do not show on the concept map.

3.1.2 Two-dimensional expression using MDS

In this paper, we use the MDS (Bryan, 1986; Hori, 1994) to identify a concept with some concept labels in the phase 2 and ii. Here, we define the standard function “Stress” to optimize the location of the concept labels. Stress is sued to minimize the deformation when multiple-dimension comes down to lower dimension.

$$Stress = \sqrt{\sum_{j=1}^{n} \frac{\sum_{i=1}^{m}\left(d_{ij} - \widehat{d_{ij}}\right)^2}{\sum_{i=1}^{m} d_{ij}^2}} \quad (6)$$

$$\hat{d}_{ij} = \frac{1}{S_{ij}}$$

where,d_{ij} is a real distance between cl_i^t and cl_j^s on the concept map. If $S_{ij} = 0$, the calculable max value is substituted for $\hat{d}_{ij}$.

If the teacher focuses on a concept label cl_j^s, then $stress_j$ shows the relationship between cl_j^s and all elements of CL^t.

$$Stress_j = \sqrt{\frac{\sum_{i=1}^{m}\left(d_{ij} - \widehat{d_{ij}}\right)^2}{\sum_{i=1}^{m} d_{ij}^2}} \quad (6)$$

In a general multiple optimization problem, the main purpose is the calculation of a global optimal solution, but we positively use the same local optimal solutions calculated by stress. Namely, we think of some solutions as some viewpoints of the teacher.

In this phase, if the teacher could not identify in spite of the two supports HM and MDS, then he/she can add and delete some concept label of the learners to let equal to the total number of concept labels between teacher and learners.

3.2 Decision of relations among the concept labels

Teacher and learners can decide the relationship between the concept labels by drawing the links between them. The teacher knows the right relationship in advance, so that teacher might draw the right partial ordering matrix. Learners are supported by the connotation algorithm in FISM. This algorithm can keep the following points;

1. Known elements are not rewritten;
2. Relations can be partially filled reachability matrix after the application of algorithm.

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Learners can finally decide the reachability matrix by taking over this algorithm.

4. EXAMINATION

In this paper, we explain the case of collaborative learning between a teacher and two learners using a sample that is the “simple classification problem of the function”. Here, the teacher indicates twelve labels and learner 1 expresses ten labels, and learner 2 expresses fourteen labels.

Phase 1 and i: Expression of the concept labels with explanatory note

In the first place, teacher and learners express the plural labels with an explanatory note concerning the function shown in table 1.

Table 1. Concept table expressed by teacher and learners

No.	Teacher	Learner 1	Learner 2
1	Function	Function	Function
2	Algebraic function	Not answer	Algebraic function
3	Logarithm Function	Logarithm Function	Logarithm Function
4	Exponential Function	Exponential Function	Exponential Function
5	Polynomial Function	Polynomial Function	Polynomial Function
6	Transcendental Function	Not answer	Transcendental function
7	Rational Function	Constant Function	Constant Function
8	Irrational Function	Identity Function	Identity Function
9	Trigonometric Function	Trigonometric Function	Trigonometric Function
10	Inverse Trigonometric Function	Inverse Trigonometric Function	Inverse Trigonometric Function
11	Injective Function	Injective Function	Injective Function
12	Surjective Function	Surjective Function	Surjective Function
13	Identity Function	Not answer	Identity Function
14	Constant Function	Not	Constant Function

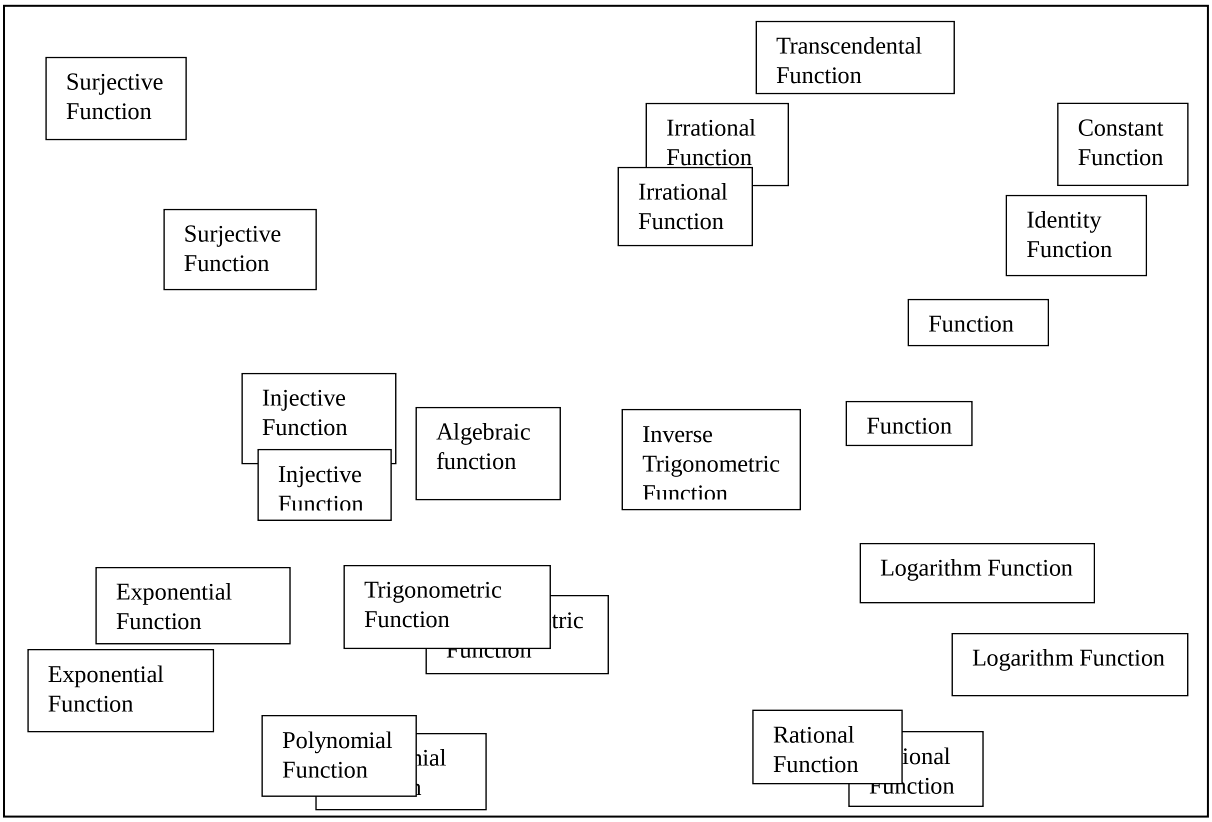

Figure 1. Unlinked concept map to identify concept labels of learners

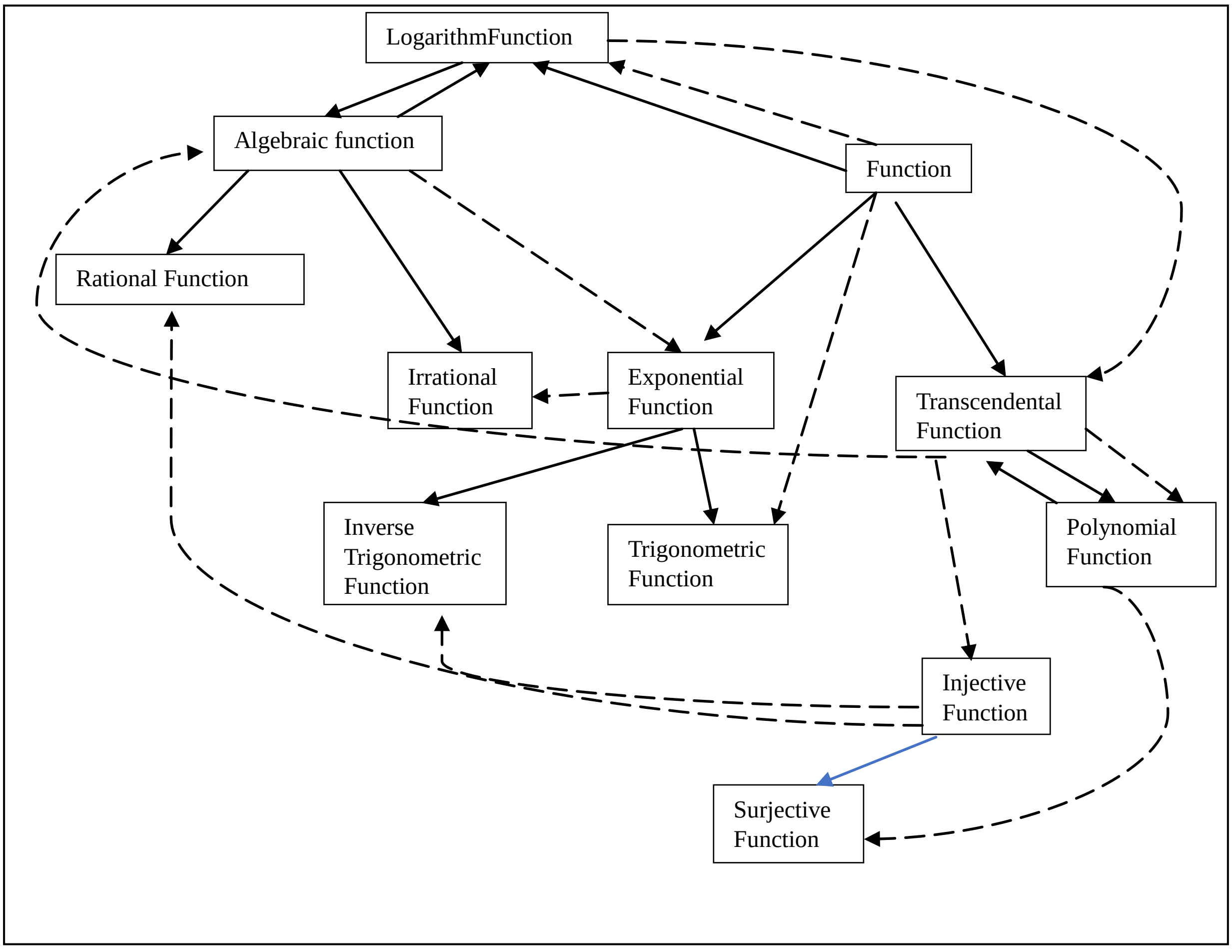

Figure 2. A concept map linked by learner 1 and learner 2

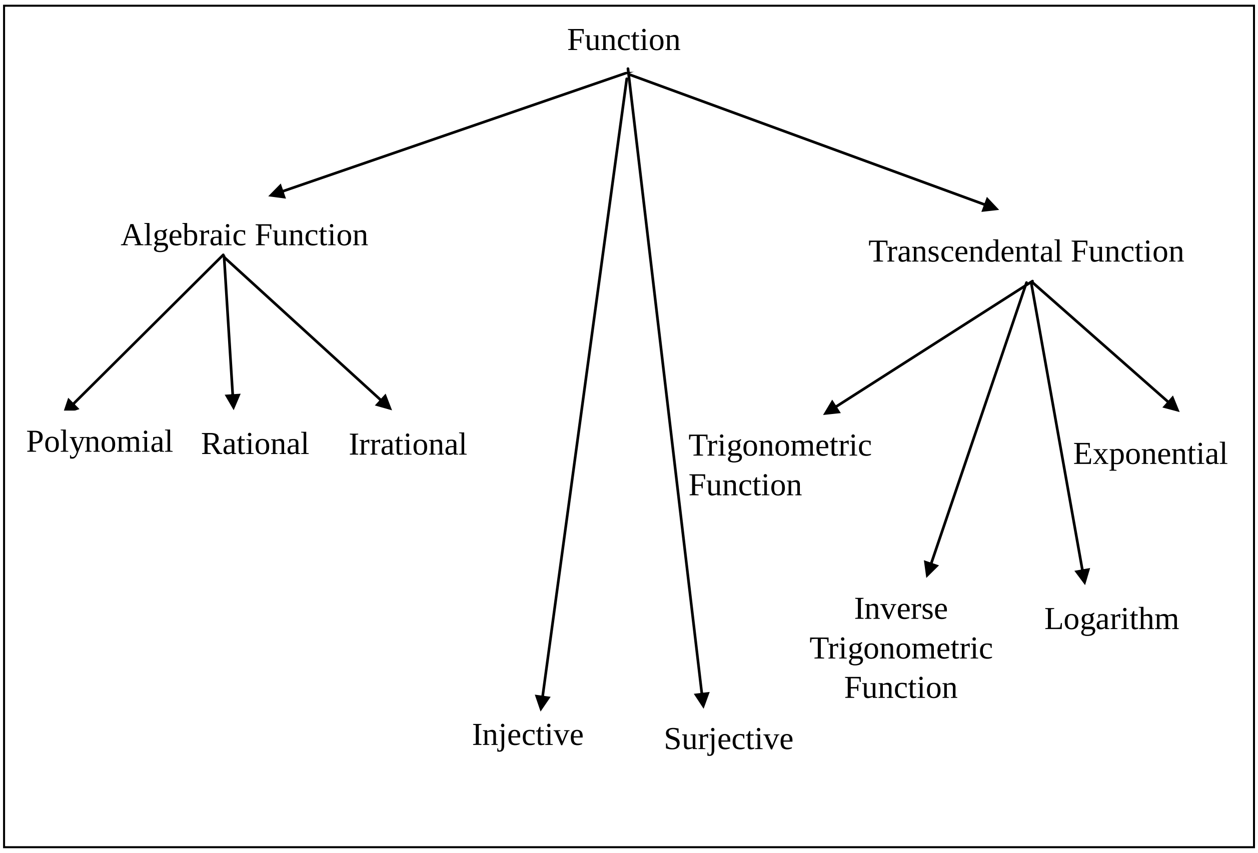

Figure 3. Concept map of Teacher

Phase 2: Identification of the concept labels among learners
Learners discuss to make common concept labels from each concept labels. They can use an unlinked concept map calculated and arranged by the MDS (Figure 1). The concept map displays concept labels of higher resemblance to near place on it.

Phase ii: Indication of the concept labels with explanatory note of teacher and learners
Next, the teacher decides the final concept labels that use the following procedure using unlinked concept map that displays concept labels of teacher and common labels of learners. The teacher returns the final labels to learners.

Phase 3: Decision of relations among the concept labels on a concept map
In this phase, each learner decides each relation among the concept labels on a shared concept map shown Figure 2. Learner 1 draws solid lines, and learner 2 illustrates with broken lines. The positions of labels are decided by discussion among the learners.

Phase iii: Decision of relations among the concept labels of teacher
Teacher also draws the concept map to decide the relations among the concept labels (Figure 3).

Phase 4: Making a consensus model among learners
We use the consensus model in FISM to make a common concept map among learners. The algorithm to make a consensus mode is following

1. Making a comparison matrix of R^1 and R^2 ;
2. While disagreement point $\neq \emptyset$ do begin;

    2.1 Calculation the consensus relation;

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2.2 Drawing opinion graph by substituting for consensus relation
2.3 Discussion about the disagreement on opinion graph for consensus.
3. End.

Here, we use follow algorithm to create comparison matrix C from plural learners.
if $S_{ij}^1 = S_{ij}^2(= S_{ij}^3)$, then $C_{ij} = S_{ij}^1 = S_{ij}^2(= S_{ij}^3)$;
else if $S_{ij}^1 \neq S_{ij}^2$ or $S_{ij}^2 \neq S_{ij}^3$ or $S_{ij}^3 \neq S_{ij}^1$, then $C_{ij} = y$.

Table 2. A comparison matrix between learner 1 and 2

Compare Matrix	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
(1) Function	1	1	1	1	1	1	1	1	1	1	1	1
(2) Algebraic function	0	1	1	0	0	0	y	1	0	0	0	0
(3) Logarithm Function	0	1	1	0	0	0	y	1	0	0	0	0
(4) Exponential Function	0	0	0	1	0	0	0	y	y	y	0	0
(5) Polynomial Function	0	0	0	0	1	y	0	0	0	0	y	1
(6) Transcendental Function	0	y	y	y	1	1	y	y	0	y	1	1
(7) Rational Function	0	0	0	0	0	0	1	0	0	0	0	0
(8) Irrational Function	0	0	0	0	0	0	0	1	0	0	0	0
(9) Trigonometric Function	0	0	0	0	0	0	0	0	1	0	0	0
(10) Inverse Trigonometric Function	0	0	0	0	0	0	0	0	0	1	0	0
(11) Injective Function	0	0	0	0	0	0	y	0	0	y	1	y
(12) Surjective Function	0	0	0	0	0	0	0	0	0	0	0	1

Table 2 describes the comparison matrix in this case. Brackets show the case of three learners.

The consensus relation is the relation that an element becomes 1 or 0 when other element of disagreement is given 1 or 0. The algorithm to calculate consensus relation is the following.

if $c_{ij} = 1$, then set $c_{ij} = 1$;

set $c_{ij} = 1$ for all $(p, q) \in CW\ 11\ (c_{ij})$;

set $c_{ij} = 0$ for all $(p, q) \in CW\ 10\ (c_{ij})$;

if $c_{ij} = 0$, then set $c_{ij} = 0$;

set $c_{ij} = 0$ for all $(p, q) \in CW\ 00\ (c_{ij})$.

Where $CW\ 11, CW\ 10$, and $CW\ 00$ are as follows.

$$CW\ 11\ (c_{ij}) = \{(p, q) | (p, q) \in Z, C_{pi}C_{jq} = 1\},$$

$$CW\ 10\ (c_{ij}) = \{(p, q) | (p, q) \in Z, C_{iq}C_{jp} + C_{qi}C_{pj} = 1\},$$

$$CW\ 00\ (c_{ij}) = \{(p, q) | (p, q) \in Z, C_{ip}C_{qj} = 1\},$$

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Figure 4 shows final consensus model among learners.

Figure 4 is an opinion graph that illustrates a minimum leading point. The arrow means that relations had to be linked, and the arrow of reverse type shows consensus relation. In this graph, the minimum leading point is a relation between that is concerned with other three relationships between “Algebraic function” and “Trigonometric function” that is concerned with other three relationships between “Logarithm function” and “Algebraic function”, “Algebraic function” and “Exponential function”, and Exponential function” and “Inverse Trigonometric function”.

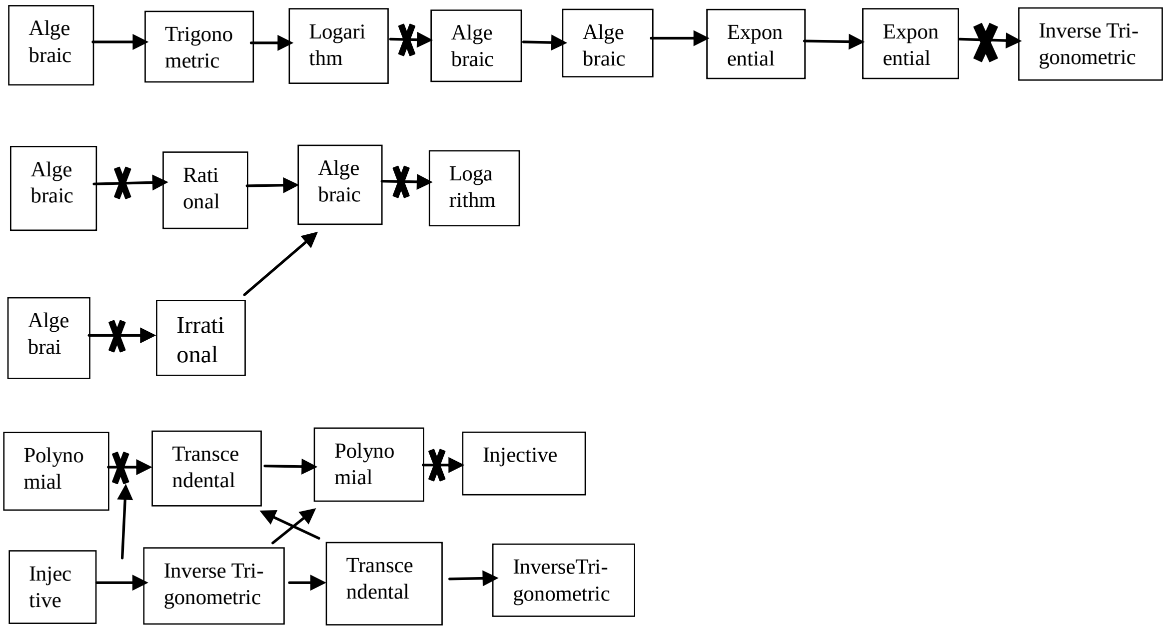

Figure 4. Opinion graph of teacher.

5. CONCLUSION AND FUTURE WORK:

In this paper, we have described a method to support collaborative learning between the teacher and multiple learners using the concept map based on FISM, and explained the learning process based on the method. By using this method, learners get some support to construct a consistent concept map of each learner, and a consensus model of learners. Teacher can know a minimum leading point from the opinion graph of teacher.

To confirm the usefulness of the proposed supporting method in collaborative learning, we have to research at least effectiveness of constructing a consensus model among learners and minimum leading point on the opinion graph of the teacher. Therefore, we consider construction and examination of the prototype system based on the proposal system based on the proposal method. In addition, to know more detailed learner’s situation of understanding, we will extend FISM to express multiple relations and fuzzy relations among concept labels on the concept map.

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